National Exams May 2014

07-Elec-A3 Signals and Communications

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an **OPEN BOOK EXAM.** Any non-communicating calculator is permitted.
3. **FIVE** (5) questions constitute a complete paper. The first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.
5. You can find a list of commonly used symbols, some trigonometric identities, Fourier Transform tables and graphs/values of Bessel functions at the end of this exam paper.

1. Consider the signal processing system in Figure (1).

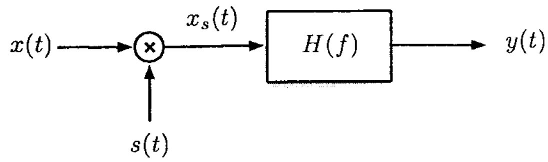


Figure 1: System block diagram.

Let the input to the system be the signal:

$$x(t) = \text{sinc}^2(\pi t), \quad (1)$$

and let $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ be a periodic impulse train with $T_s = 1/4$ seconds.

(a) Determine and sketch $X(f) = \mathcal{F}[x(t)]$.

(b) Determine the Nyquist frequency of $x(t)$.

(c) We sample $x(t)$ by multiplying with $s(t)$. Determine the spectrum of the sampled waveform $X_s(f) = \mathcal{F}[x_s(t)] = \mathcal{F}[x(t)s(t)]$. Sketch $X_s(f)$ for $|f| < 10$ Hz.

(d) Let $H(f)$ be an ideal lowpass filter with frequency response shown in Figure (2). Determine the filter gain A and the allowable range for f_c such that $y(t) = x(t)$.

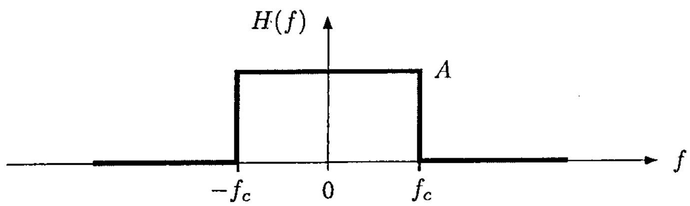


Figure 2: Frequency response $H(f)$.

2. The baseband message signal $x(t)$

$$x(t) = 2\cos(2\pi 1000\, t)\cos(2\pi 2000\, t) \tag{2}$$

is sampled by an ***ideal sampler*** at a rate of f_s–Hz.

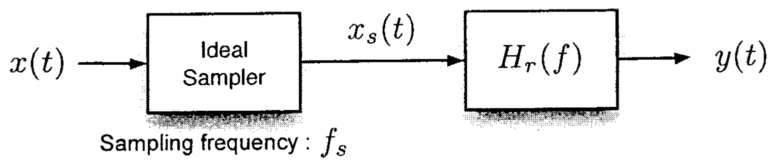


Figure 3: Generating $x_s(t)$ and $y(t)$ from $x(t)$.

The sampled waveform $x_s(t)$ is subsequently converted back to continuous-time domain by post-filtering with the recovery filter $H_r(f)$. Assume $H_r(f)$ is an ideal lowpass filter with unity gain, zero-phase and cut-off frequency $f_s/2$.

(a) Determine and plot $X(f)$, the spectrum of the message signal $x(t)$.

(b) Determine f_{Nyq}, the Nyquist rate of the signal $x(t)$.

(c) If $f_s = 2f_{Nyq}$, plot $X_s(f)$ and $Y(f)$ for $|f| \leq 10$ kHz. Also determine $y(t)$.

(d) If $f_s = \frac{5}{6}f_{Nyq}$, plot $X_s(f)$ and $Y(f)$ for $|f| \leq 10$ kHz. Also determine $y(t)$.

(e) Compare the $y(t)$ results obtained in parts (c) and (d) with the message signal $x(t)$, and comment on observed differences.

3. Let $x(t) = 2\,\text{sinc}(2\pi t)$.

   (a) Determine $X(f) = \mathcal{F}\big[x(t)\big]$.

   (b) Sketch and label the **magnitude** and **phase spectra** of $y_1(t) = x\big(2(t-0.5)\big)$.

   (c) Sketch and label the **magnitude** and **phase spectra** of $y_2(t) = x\big(2(t-0.5)\big)\cos 10\pi t$.

4. Let $m(t)$ be the single tone signal:

$$m(t) = A_m \cos 2\pi f_m t. \tag{3}$$

$m(t)$ is used as the input to a frequency modulator which generates the frequency modulated (FM) signal:

$$\varphi_{\text{FM}}(t) = A_c \cos\big(2\pi f_c t + 2\pi K_f \int_0^t m(\lambda) d\lambda\big) \tag{4}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\big(2\pi(f_c + n f_m)t\big) \tag{5}$$

with $A_c = 100$ V. In Equation (5), $J_n(\beta)$ is the Bessel function of $1st$ kind, order-n with argument β, and $\beta = \Delta f / f_m$ is the modulation index of the FM signal $\varphi_{\text{FM}}(t)$. Figure (4) shows the magnitude spectrum $|\Phi_{\text{FM}}(f)| = |\mathcal{F}[\varphi_{\text{FM}}(t)]|$ for $f \geq 0$.

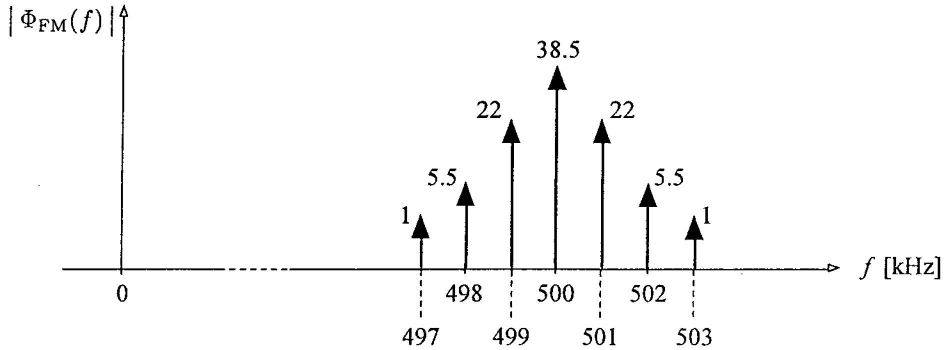


Figure 4: Magnitude spectrum $|\Phi_{\text{FM}}(f)|$ (shown for $f \geq 0$ only).

(a) Determine the peak frequency deviation Δf_{peak}, f_m and f_c of the FM signal $\varphi_{\text{FM}}(t)$ (a plot of Bessel functions as well as a table of values of Bessel functions is provided at the end of this exam paper).

(b) If A_m is 2.0 V, determine the modulation sensitivity parameter K_f.

(c) Sketch the magnitude spectrum of the modulated output, if A_m is increased to 4.0 V and f_m is increased by a factor of 2.

5. A television signal (video and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized and binary-coded to obtain a PCM signal.

   (a) Determine the sampling rate if the signal to be sampled at a rate 20% above the Nyquist rate.

   (b) If the samples are quantized into 1024 levels, determine the number of binary pulses required to encode each sample.

   (c) Determine the binary pulse rate (bits per second) of the binary-coded signal and the minimum bandwidth required to transmit the signal.

6. Consider the AM signal $\varphi_{\text{AM}}(t) = [A_c + m(t)] \cos 2\pi f_c t$, where $m(t)$ is the real-valued modulating signal with the spectrum $\Phi_{\text{AM}}(f) = \mathcal{F}[\,\varphi_{\text{AM}}(t)\,]$ as shown in Figure (5).

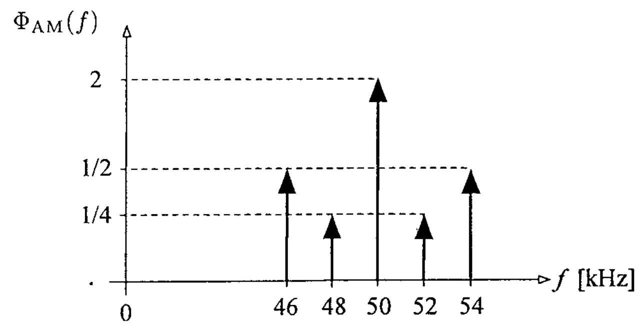


Figure 5: Spectrum of the AM signal (*shown only for* $f \geq 0$).

(a) Determine f_c, A_c and $m(t)$.

(b) Determine the modulation index μ of the AM signal.

(c) Determine the average sideband power P_s, the average carrier power P_c, the average total signal power $P_{\varphi_{\text{AM}}}$ and the power efficiency η of $\varphi_{\text{AM}}(t)$ (assume the load with respect to all power values are measured is normalized to unity).

(d) If $m(t)$ in $\varphi_{\text{AM}}(t)$ is to be replaced by $Km(t)$ where $K \in \mathbb{R}^+$, determine the maximum value of K such that we can recover $m(t)$ from $\varphi_{\text{AM}}(t)$ using an envelope detector. Also determine the power efficiency using the value of K you have determined. How does it compare with the power efficiency you calculated in part (c)?

7. Let $x(t) = 2\cos 2\pi f_0 t$ be the input to a transmission system shown in Figure (6).

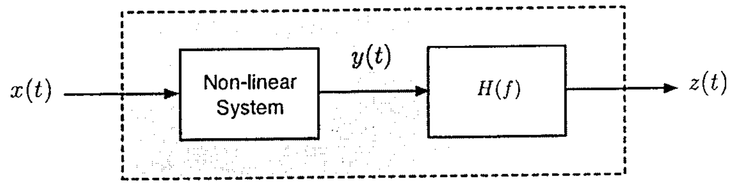


Figure 6: A transmission system.

The non-linear system is characterized by the input-output relation:

$$[\text{output}] = [\text{input}] + [\text{input}]^2. \tag{6}$$

$H(f)$ represents a lowpass filter with the frequency response shown in Figure (7):

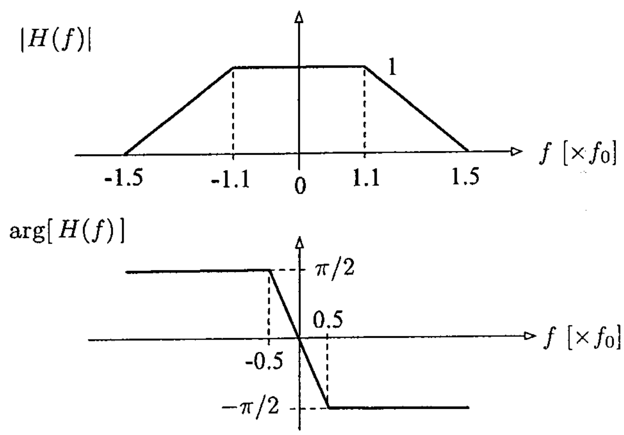


Figure 7: Magnitude and phase spectra of $H(f)$.

(a) Determine and sketch $X(f)$ and $Y(f)$.

(b) Sketch $Z(f)$ and determine $z(t)$.

(c) You are asked to make an assessment about the characteristics of the cascade system shown in the dashed box, based **only** on the result you obtained in part (b), i.e., by looking at the input signal $x(t) = 2\cos 2\pi f_0 t$ and the output signal $z(t)$ determined in part (b). Based on $x(t)$ and $z(t)$ you observed/calculated, does the cascade system shown within the dashed box represent: (i) a linear system, (ii) a distortionless system? Please explain and justify your answers.

(d) If you were not allowed to see what is inside the dashed box, how would you design experiments to determine whether the entire system represented by the dashed box: (i) is a linear system or not, and (ii) is a distortionless system or not? Please explain the rationale of the design of your experiments.

8. Let $H(z)$ given in Equation (7) be the transfer function of a discrete-time, linear, time-invariant system:

$$H(z) = \frac{z\,(z-1)}{(z-0.5)\,(z-2)}. \tag{7}$$

(a) Identify all possible *regions-of-convergence* (ROC) applicable to this system.

(b) Determine the system characteristics (*causality*, *stability*) corresponding to each *region-of-convergence* identified in part (a). Explain and justify your answers.

(c) Determine the impulse response sequence $h[n]$ corresponding to each *region-of-convergence* identified in part (a). Prove/show that the system characteristics identified in part (b) can also be derived from the impulse response sequences.

(d) Find the difference equation that characterizes the system.

9. An audio signal of bandwidth 3.2 kHz is sampled at a rate 25% above the Nyquist rate and quantized by a uniform quantize. The quantization error is not to exceed 0.5% of the peak signal amplitude. The resulting quantized samples are now coded and transmitted by 4-ary pulses.

   (a) Determine the number of 4-ary pulses required to encode each sample.

   (b) Determine the minimum transmission bandwidth required to transmit this data with zero inter-symbol interference (ISI).

   (c) Determine the transmission bandwidth if Nyquist criterion 4-ary pulses with 25% roll-off factor are used to transmit the data.

10. Consider the split-phase Manchester format, where

$$p(t) = \begin{cases} 1, & 0 < t < \frac{T_b}{2}; \\ -1, & \frac{T_b}{2} < t < T_b. \end{cases} \tag{8}$$

(a) Determine and plot $h(t)$, the impulse response function of the corresponding **causal** matched filter.

(b) Use superposition to plot the pulse response $A\,p(t - T_b) * h(t)$.

(c) A noisy, split-phase Manchester formatted waveform with $a_k = \pm 1$ V and $r_b = 1$ kbps has been received and filtered by the matched filter with the impulse response as in part (a). Figure (8) depicts the eye diagram and the waveform at the output of the matched filter, respectively.

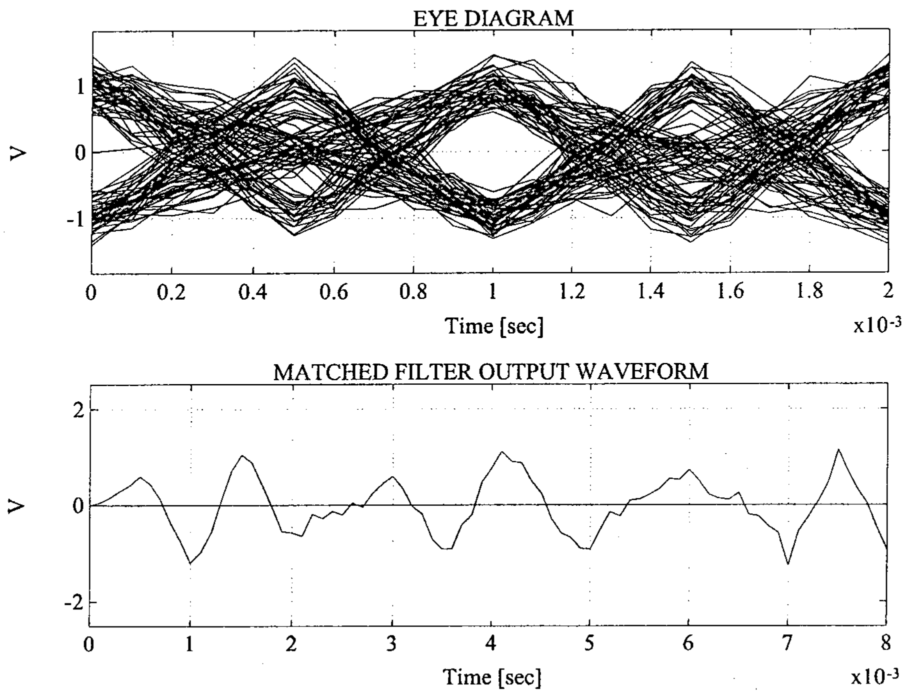


Figure 8: **(a)** Eye diagram, **(b)** time-waveform at the output of the matched filter.

Determine the optimum sampling instants and the optimum decoding rule. Apply these results to decode the first six message symbols from the matched filter output waveform. Indicate the optimum sampling instants and the sample values used by the decoder on the figure.

Trigonometric Identities:

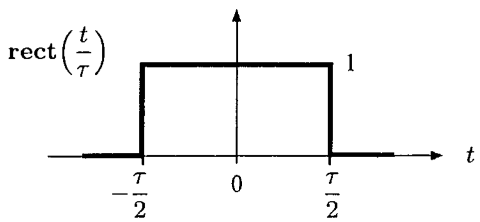
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin\alpha\,\sin\beta = \tfrac{1}{2}\cos(\alpha-\beta) - \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\cos\alpha\,\cos\beta = \tfrac{1}{2}\cos(\alpha+\beta) + \tfrac{1}{2}\cos(\alpha-\beta)$$
$$\sin\alpha\,\cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta)$$

Euler's Identity:

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad \cos\alpha = \frac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right) \qquad \sin\alpha = \frac{1}{2j}\left(e^{j\alpha} - e^{-j\alpha}\right)$$

Rectangular and Triangular Pulse Functions:



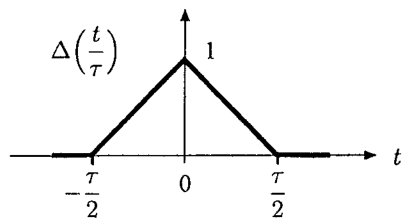


Table 1: Properties of Fourier Transform Operations

Operation	$g(t)$	$G(f)$
Superposition	$g_1(t) + g_2(t)$	$G_1(f) + G_2(f)$
Scalar multiplication	$kg(t)$	$kG(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$
Time shifting	$g(t-t_0)$	$G(f)\,e^{-j2\pi f t_0}$
Frequency shifting	$g(t)\,e^{j2\pi f_0 t}$	$G(f-f_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(f)G_2(f)$
Frequency convolution	$g_1(t)g_2(t)$	$G_1(f) * G_2(f)$
Time differentiation	$\frac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^{t} g(x)dx$	$\frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$

Table 2: Short Table of Fourier Transforms

$g(t)$	$G(f)$
$e^{-at}u(t)$	$[a+j2\pi f]^{-1}, \quad a>0$
$e^{at}u(-t)$	$[a-j2\pi f]^{-1}, \quad a>0$
$e^{-a\|t\|}$	$2a\,[a^2+(2\pi f)^2]^{-1}, \quad a>0$
$te^{-at}u(t)$	$[a+j2\pi f]^{-2}, \quad a>0$
$t^n e^{-at}u(t)$	$n!\,[a+j2\pi f]^{-(n+1)}, \quad a>0$
$\delta(t)$	1
1	$\delta(f)$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$
$\cos 2\pi f_0 t$	$\frac{1}{2}\big[\delta(f+f_0)+\delta(f-f_0)\big]$
$\sin 2\pi f_0 t$	$\frac{j}{2}\big[\delta(f+f_0)-\delta(f-f_0)\big]$
$u(t)$	$\frac{1}{2}\delta(f)+[j2\pi f]^{-1}$
$\mathrm{sgn}(t)$	$2[j2\pi f]^{-1}$
$\cos(2\pi f_0 t)\,u(t)$	$\frac{1}{4}\big[\delta(f-f_0)+\delta(f+f_0)\big]+j2\pi f\big[(2\pi f_0)^2-(2\pi f)^2\big]^{-1}$
$\sin(2\pi f_0 t)\,u(t)$	$\frac{1}{4j}\big[\delta(f-f_0)-\delta(f+f_0)\big]+2\pi f_0\big[(2\pi f_0)^2-(2\pi f)^2\big]^{-1}$
$e^{-at}\sin(2\pi f_0 t)\,u(t)$	$2\pi f_0\big[(a+j2\pi f)^2+(2\pi f_0)^2\big]^{-1}, \quad a>0$
$e^{-at}\cos(2\pi f_0 t)\,u(t)$	$\big[a+j2\pi f\big]\big[(a+j2\pi f)^2+(2\pi f_0)^2\big]^{-1}, \quad a>0$
$\mathbf{rect}(\frac{t}{\tau})$	$\tau\,\mathrm{sinc}(\pi f\tau)$
$2B\,\mathrm{sinc}(2\pi Bt)$	$\mathbf{rect}(\frac{f}{2B})$
$\Delta(\frac{t}{\tau})$	$\frac{\tau}{2}\,\mathrm{sinc}^2\left(\frac{\pi f\tau}{2}\right)$
$B\,\mathrm{sinc}^2(\pi Bt)$	$\Delta(\frac{f}{2B})$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0), \quad f_0=1/T_0$
$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-2(\sigma\pi f)^2}$

Table 3: Some Common z-Transform Pairs

Sequence	Transform	Region-of-Convergence
$\delta[n]$	1	all z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	all z except 0 (if $m > 0$)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$\sin(\omega_0 n)u[n]$	$\frac{\sin(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\|z\| > 1$
$r^n\cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$r^n\sin(\omega_0 n)u[n]$	$\frac{r\sin(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\|z\| > r$
$\begin{cases} a^n, & 0 \le n \le N-1; \\ 0, & \text{otherwise.} \end{cases}$	$\frac{1-a^N z^{-N}}{(1-az^{-1})^2}$	$\|z\| > 0$

In the above table $\delta[n]$ is the unit sample sequence:

$$\delta[n] = \begin{cases} 1, & n = 0; \\ 0, & n \neq 0; \end{cases}$$

and $u[n]$ is the unit-step sequence:

$$u[n] = \begin{cases} 1, & n = 0, 1, 2, 3, \ldots \\ 0, & n < 0. \end{cases}$$

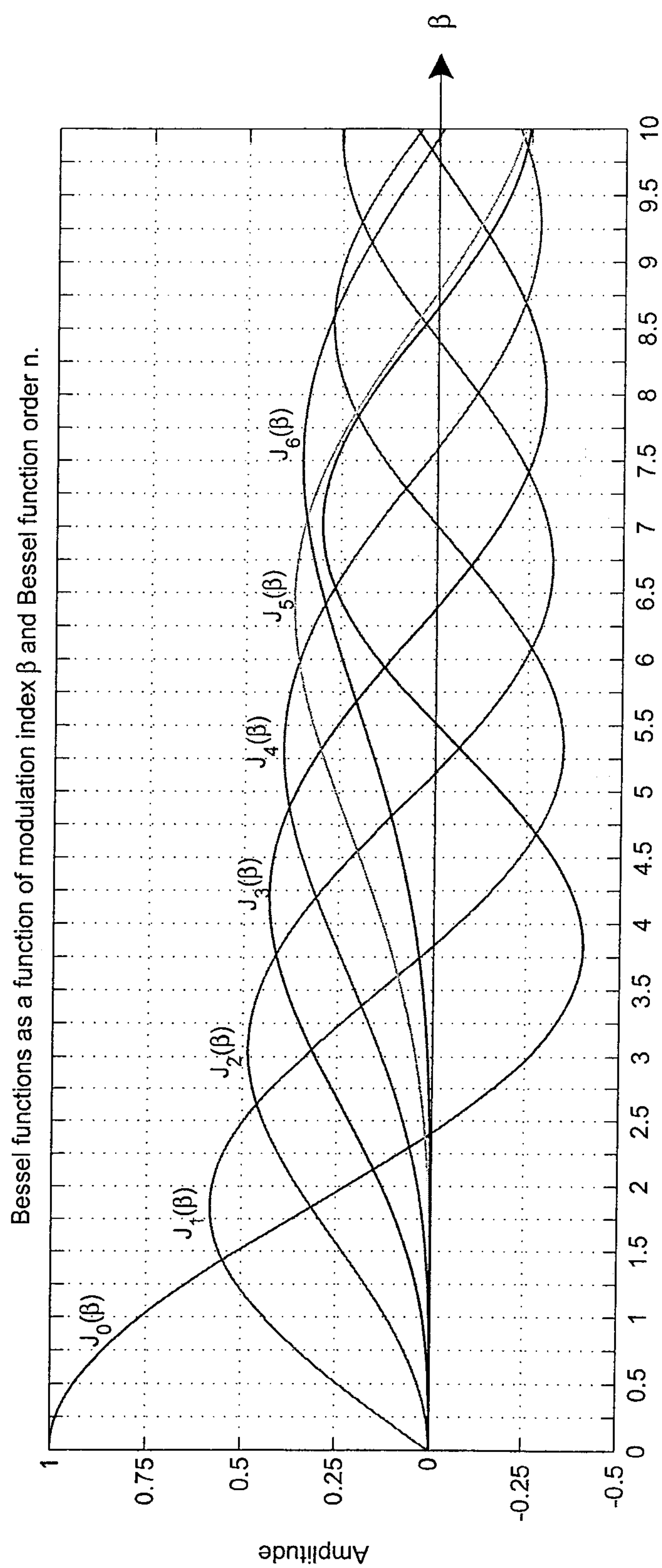

Bessel functions as a function of modulation index β and Bessel function order n.
$J_0(\beta)$
$J_1(\beta)$
$J_2(\beta)$
$J_3(\beta)$
$J_4(\beta)$
$J_5(\beta)$
$J_6(\beta)$
Amplitude
β
1
0.75
0.5
0.25
0
-0.25
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10

Table of Bessel Functions as a function of modulation index β and Bessel function order n.

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$J_3(\beta)$	$J_4(\beta)$	$J_5(\beta)$	$J_6(\beta)$	$J_7(\beta)$	$J_8(\beta)$	$J_9(\beta)$
0.0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.99	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.4	0.96	0.20	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.91	0.29	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.8	0.85	0.37	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00
1.0	0.77	0.44	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00
1.2	0.67	0.50	0.16	0.03	0.01	0.00	0.00	0.00	0.00	0.00
1.4	0.57	0.54	0.21	0.05	0.01	0.00	0.00	0.00	0.00	0.00
1.6	0.46	0.57	0.26	0.07	0.01	0.00	0.00	0.00	0.00	0.00
1.8	0.34	0.58	0.31	0.10	0.02	0.00	0.00	0.00	0.00	0.00
2.0	0.22	0.58	0.35	0.13	0.03	0.01	0.00	0.00	0.00	0.00
2.2	0.11	0.56	0.40	0.16	0.05	0.01	0.00	0.00	0.00	0.00
2.4	0.00	0.52	0.43	0.20	0.06	0.02	0.00	0.00	0.00	0.00
2.6	-0.10	0.47	0.46	0.24	0.08	0.02	0.01	0.00	0.00	0.00
2.8	-0.19	0.41	0.48	0.27	0.11	0.03	0.01	0.00	0.00	0.00
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	0.00	0.00	0.00
3.2	-0.32	0.26	0.48	0.34	0.16	0.06	0.02	0.00	0.00	0.00
3.4	-0.36	0.18	0.47	0.37	0.19	0.07	0.02	0.01	0.00	0.00
3.6	-0.39	0.10	0.44	0.40	0.22	0.09	0.03	0.01	0.00	0.00
3.8	-0.40	0.01	0.41	0.42	0.25	0.11	0.04	0.01	0.00	0.00
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	0.00	0.00
4.2	-0.38	-0.14	0.31	0.43	0.31	0.16	0.06	0.02	0.01	0.00
4.4	-0.34	-0.20	0.25	0.43	0.34	0.18	0.08	0.03	0.01	0.00
4.6	-0.30	-0.26	0.18	0.42	0.36	0.21	0.09	0.03	0.01	0.00
4.8	-0.24	-0.30	0.12	0.40	0.38	0.23	0.11	0.04	0.01	0.00
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	0.01
5.2	-0.11	-0.34	-0.02	0.33	0.40	0.29	0.15	0.07	0.02	0.01
5.4	-0.04	-0.35	-0.09	0.28	0.40	0.31	0.18	0.08	0.03	0.01
5.6	0.00	-0.33	-0.15	0.23	0.39	0.33	0.20	0.09	0.04	0.01
5.8	0.09	-0.31	-0.20	0.17	0.38	0.35	0.22	0.11	0.05	0.02
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02

Marking Scheme

Question 1:

(a) 3, **(b)** 3, **(c)** 7, **(d)** 7 marks.

Question 2:

(a) 4, **(b)** 4, **(c)** 4, **(d)** 4, **(e)** 4 marks.

Question 3:

(a) 3, **(b)** 8, **(c)** 9 marks.

Question 4:

(a) 7, **(b)** 7, **(c)** 6 marks.

Question 5:

(a) 5, **(b)** 7, **(c)** 8 marks.

Question 6:

(a) 3, **(b)** 3, **(c)** 7, **(d)** 7 marks.

Question 7:

(a) 5, **(b)** 5, **(c)** 5, **(d)** 5 marks.

Question 8:

(a) 5, **(b)** 5, **(c)** 5, **(d)** 5 marks.

Question 9:

(a) 6, **(b)** 7, **(c)** 7 marks.

Question 10:

(a) 7, **(b)** 7, **(c)** 6 marks.