National Exams May 2018

16-Elec-A2, Systems and Control

3 hours duration

NOTES:

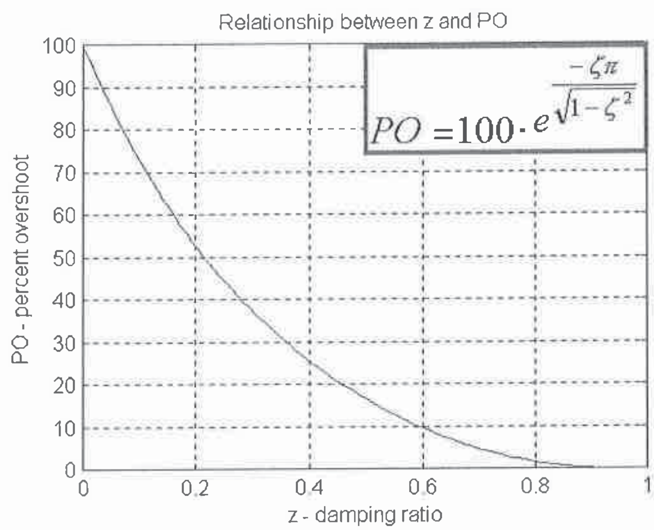
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

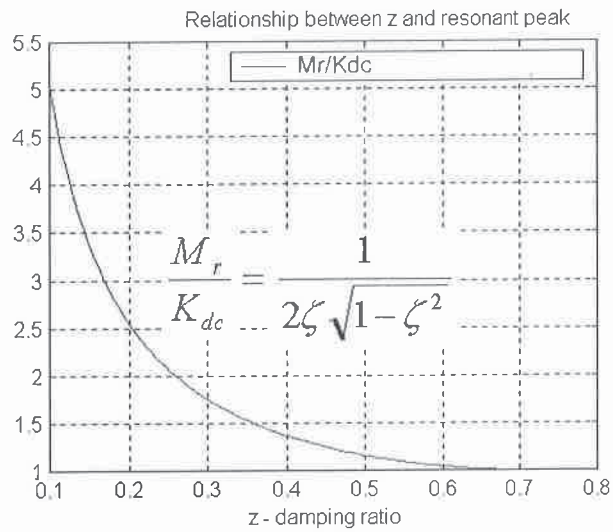
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t+\cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s)-f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

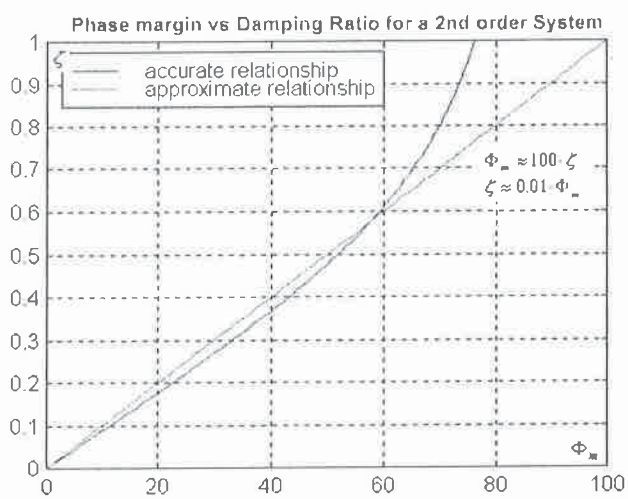
Useful Plots & Formulae



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio



Second Order Model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Definitions for Controllability Matrix, $\mathbf{M_c}$, and Observability Matrix, $\mathbf{M_o}$:

$$\mathbf{M}_c = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B}\end{bmatrix} \qquad \mathbf{M}_o = \begin{bmatrix}\mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2\end{bmatrix}$$

Definition for Transfer Function from State Space:

$$\mathrm{G(s)} = \mathbf{C} \cdot (\mathrm{s}\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$$

Question 1 (Compulsory)

Transfer Functions - Signal Flow Diagrams – Mason's Gain Formula. Stability – Routh Array and Routh-Hurwitz Criterion of Stability. Steady State Errors.

Consider the block diagram of the servo-control system for one of the joints of a robot arm, shown in Figure Q1.1, where the input is the reference angular velocity (speed) for the robot arm, $\Omega_{ref}(s)$, the output is the actual load velocity of the arm, $\Omega_{load}(s)$, and the forward path contains a Proportional + Integral (PI) Controller, a calibration gain, motor and robotic arm dynamics and a gearbox. The Proportional + Integral (PI) Controller is described as shown, with the Integral Time constant, τ_i, equal to 0.1 seconds:

$$G_c(s) = K_p\left(1 + \frac{1}{\tau_i s}\right)$$

Two transfer functions have been defined, the closed loop system transfer function $G_{cl}(s)$, between the actual angular load velocity and the reference angular velocity, and the disturbance transfer function, $G_{dist}(s)$, between the Torque disturbance signal, $T_d(s)$, and the output speed, $\Omega_{load}(s)$:

$$G_{cl}(s) = \frac{\Omega_{load}(s)}{\Omega_{ref}(s)} \qquad\qquad G_{dist}(s) = \frac{\Omega_{load}(s)}{T_d(s)}$$

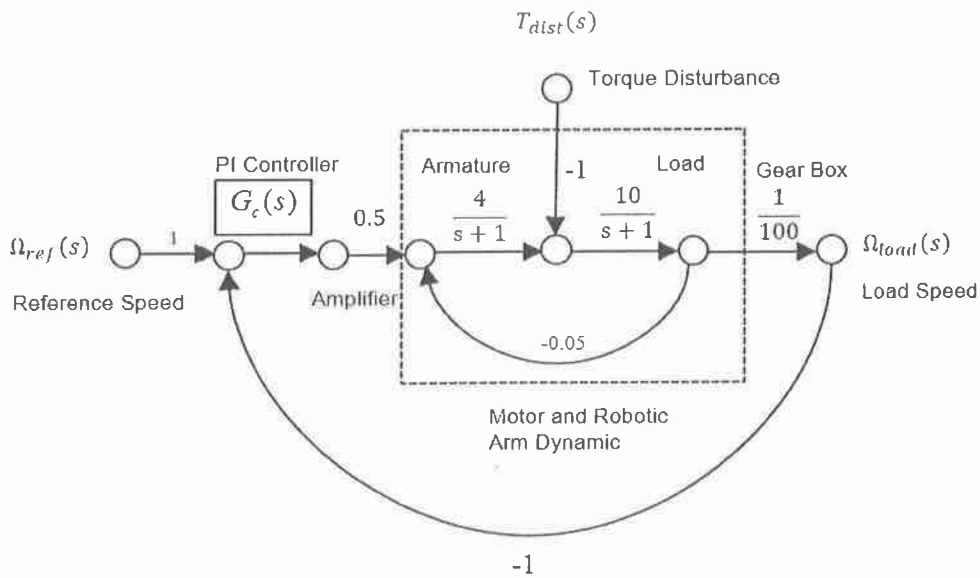


Figure Q1.1: Servomotor Positioning System

1) **(7 marks)** Find $G_{cl}(s)$ and write it in the TF format (polynomial ratio), as a function of the Proportional gain, K_p.

2) **(3 marks)** Find $G_d(s)$ and write it in the TF format (polynomial ratio), as a function of the Proportional gain, K_p.

3) **(10 marks)** Determine the range of the Proportional Controller gains, K_p, for a safe, stable operation of the closed loop system. Specify the critical value(s) of the Gain, K_{crit}, when the system is marginally stable, as well as the frequency of oscillations, ω_{osc}, resulting when $K_p = K_{crit}$.

Question 2 (Compulsory)

Second Order Dominant Poles Model, System Performance.

Consider, once again, the servo-positioning system under Proportional + Integral (PI) Control from Question 1, shown in Figure Q1.1. No disturbance signal is present. The Controller Operational Gain value was set to $K_{op} = 3.0$, and then the system open loop and closed loop transfer functions, $G_{open}(s)$ and $G_{cl}(s)$, were found to be as follows:

$$G_{open}(s) = \frac{0.6(s+10)}{s(s^2+2s+3)} \qquad G_{cl}(s) = \frac{0.6(s+10)}{(s+1.827)(s^2+0.173s+3.284)}$$

1) **(6 marks)** Calculate the system Error Constants and corresponding steady state errors: K_{pos}, K_v, K_a, $e_{ss(step)\%}$, $e_{ss(ramp)}$, $e_{ss(parab)}$.
2) **(2 marks)** What is the DC gain of the closed loop system, $G_{cl}(0)$?
3) **(8 marks)** Assume that a 2nd order dominant poles model (see page 3) will be applicable for the closed loop system. Determine the model parameters, $K_{dc}, \zeta, \omega_n,$ and write the model transfer function, $G_m(s)$.
4) **(4 marks)** Evaluate the following unit step response specifications: Percent Overshoot (PO), Settling Time ($T_{settle\ \pm 2\%}$) and Rise Time ($T_{rise\ (0-100\%)}$).

Question 3

Steady State Error Analysis, Final Value Theorem, Superposition Theorem.

Consider, once again, the same servo-positioning system under Proportional + Integral (PI) Control from Question 1, shown in Figure Q1.1. The system is to exhibit zero steady state error to a unit step and the steady state error to a unit ramp is to be: $e_{ss(ramp)} = 0.5\ V/V$.

1) **(8 marks)** Calculate the required operating value for the Proportional Gain (K_{op}).
2) **(4 marks)** When $K_p = K_{op}$, will the system be still stable? Calculate the corresponding Gain Margin (G_m), in V/V units.
3) **(5 marks)** First, assume the Proportional Controller Gain to be equal to one: $K_p = 1$. Assume the reference angular velocity and the disturbance torque signals to be described as show below, and then calculate the total steady state error in the system response.

$$\omega_{ref} = 2t \cdot 1(t) \qquad T_{dist}(t) = 10t \cdot 1(t)$$

4) **(3 marks)** Next, assume the Proportional Controller Gain to be equal to the Operational Gain as calculated in item 1) above: $K_p = K_{op}$. Repeat the calculation of the total steady state error in the system response.

Question 4

State Space Model from Transfer Functions, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear open loop system described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

1) **(4 marks)** Find the system eigenvalues. Is the open loop system stable?
2) **(4 marks)** Find the open loop system transfer function, $G(s) = \dfrac{Y(s)}{U(s)}$.
3) **(4 marks)** Determine if the open loop system is observable and/or controllable.
4) **(4 marks)** Place the system in a closed loop configuration with the reference input **r** and assume the controller equation to be in the form:

$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

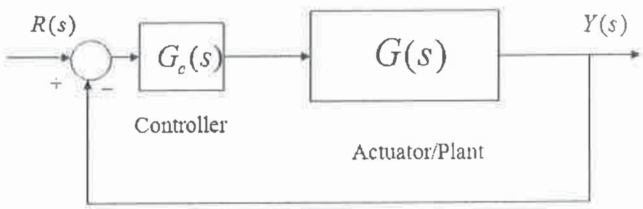
Determine the values of the Proportional Gain K and the state feedback vector gains $\mathbf{k}$ so that the closed loop system will have poles at: -3 and -4, and the steady-state error to a step input will be zero.

5) **(4 marks)** Find the closed loop system transfer function, $G_{cl}(s) = \dfrac{Y(s)}{R(s)}$

Question 5

Controller Design in Frequency Domain – Lead Controller, Second Order Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown below.



The system is to operate under Lead Control. The Lead Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau s + 1}{\alpha \tau s + 1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lead Time Constant and $\alpha < 1$.

The process transfer function G(s) is as follows:

$$G(s) = \frac{1}{s(s+1)(s+2.7)}$$

The uncompensated Open Loop Frequency Response plot is shown in Figure Q5.1. The design requirements are:

- The Steady State Error for the unit ramp input for the compensated closed loop system is to be equal to 0.27 V/V;
- Percent Overshoot of the compensated closed loop system is to be no more than 25%;
- The Settling Time, $T_{settle(\pm 2\%)}$, is to be no more than 2.5 seconds.

1) **(2 marks)** Calculate the Position Constant for the uncompensated system (K_{pos_u}), then the Position Constant for the compensated system (K_{pos_c}) that would meet the design requirements.
2) **(4 marks)** Read off the Phase Margin (Φ_{m_u}) and the crossover frequency (ω_{cp_u}) of the uncompensated system. Next, decide what their values should be ($\Phi_{m_c}, \omega_{cp\_c}$) so that the compensated system meets the design requirements.
3) **(10 marks)** Calculate the appropriate Lead Controller parameters and the Controller transfer function.
4) **(4 marks)** Finally, estimate the compensated closed loop step response specs: $e_{ss(step\%)}$, $e_{ss(ramp)}$, $T_{settle(\pm 2\%)}$ and PO.

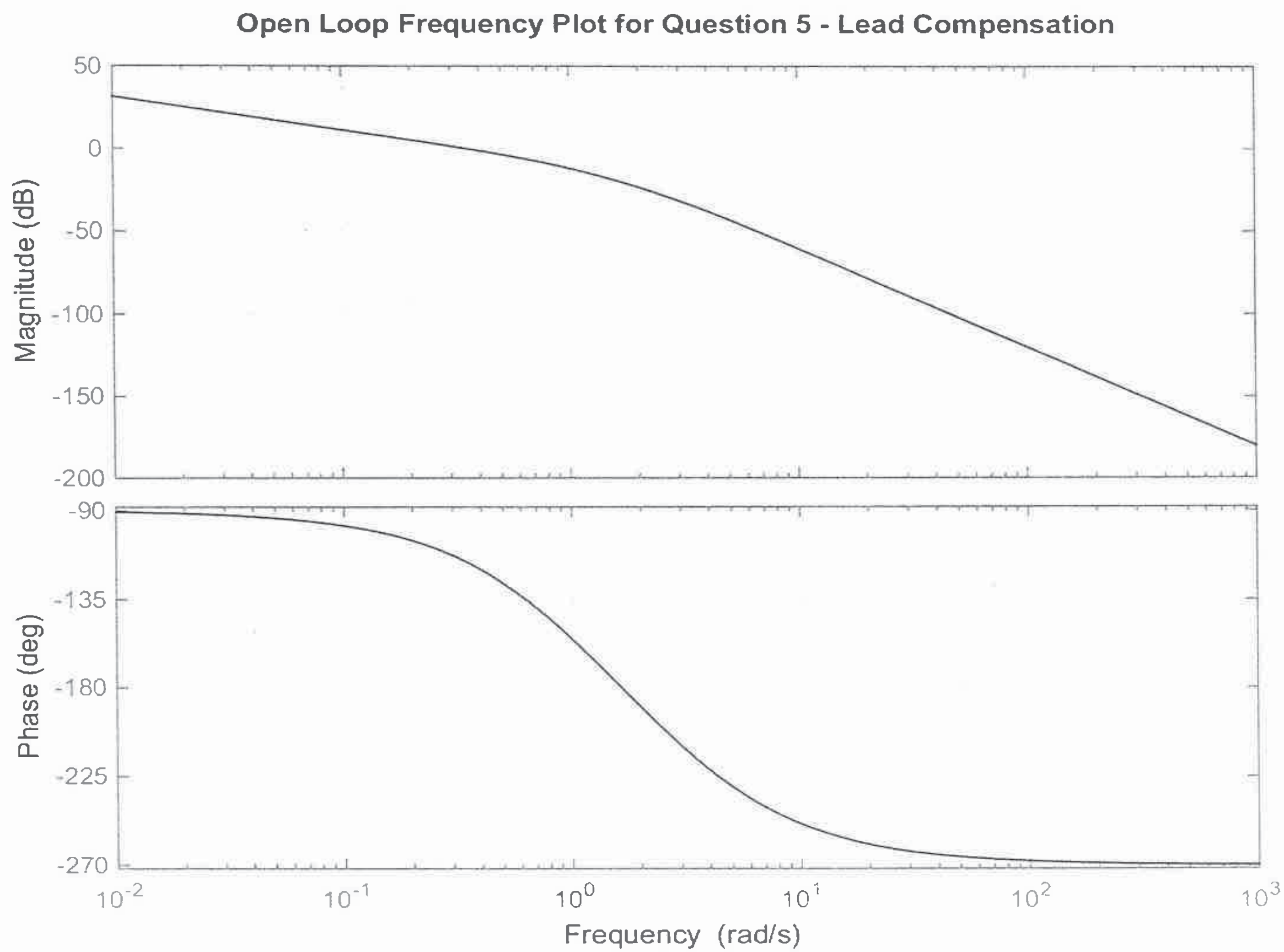
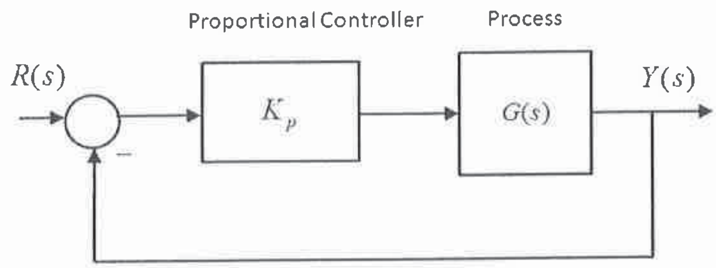


Figure Q5.1 - Open Loop Frequency Response Plots for Lead Design

Question 6

Stability, Root Locus Analysis, Second Order Dominant Poles Model.

A unit feedback control system is working under a Proportional Controller, as shown below.



The process transfer function is described as follows:

$$G(s) = \frac{1}{(s-5)(s^2+6s+40)}$$

1) **(8 marks)** Determine the value(s) of the Proportional Controller Gain, $K_p = K_{crit}$, at which the closed loop system becomes marginally stable, and the corresponding frequency(ies) of marginally stable oscillations, ω_{osc}.
2) **(8 marks)** In the space provided in Figure Q6.1, sketch a detailed Root Locus for the system, including break-away/break-in coordinates, asymptotes, angles of departure from complex poles, a centroid, etc. If you are using estimates, explain why.
3) **(4 marks)** Is it possible to assume that the closed loop system can be approximated by a second order model so that the equivalent damping ratio is $\zeta = 0.707$? Discuss briefly.

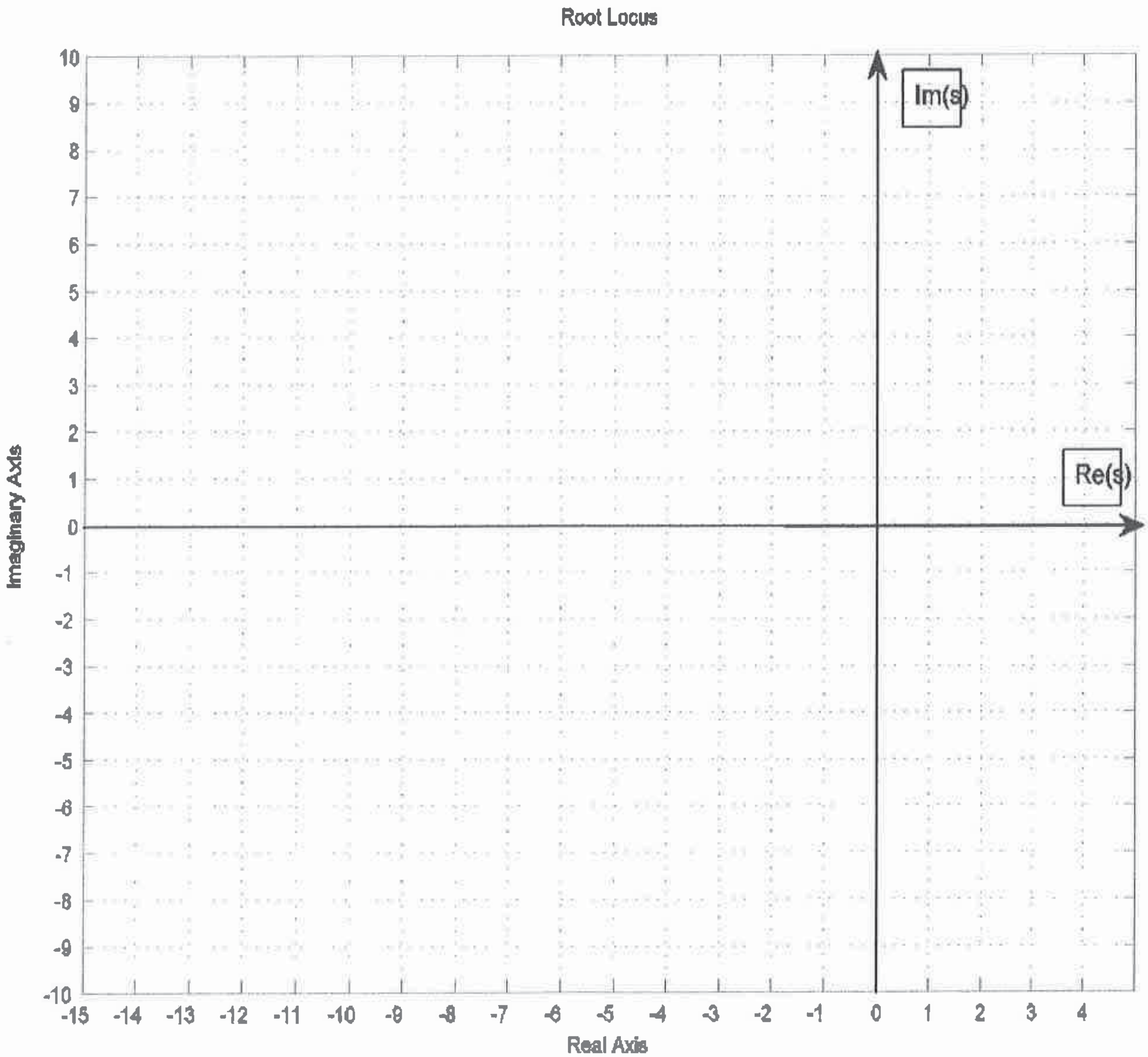


Figure Q6.1 – Place Your Root Locus Graph Here

Question 7

PID Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider two versions of a closed loop system, working under Proportional + Rate Feedback Control, and Proportional + Derivative Control, shown in Figures Q7.1 and Q7.2, respectively.

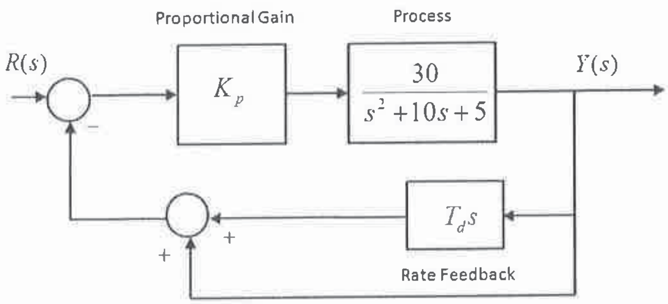


Figure Q7.1

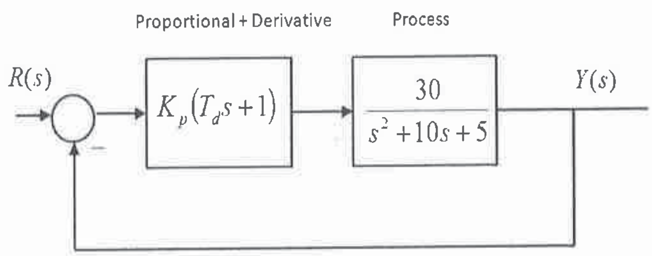


Figure Q7.2

1) **(10 marks)** The compensated Closed Loop step response of this system is to have the following specifications: Percent Overshoot $PO = 5\%$ and Steady State Error $e_{step(\%)} = 5\%$. Determine the Controller settings for the Proportional + Rate Feedback Control (K_p and T_d) to meet these requirements.

2) **(4 marks)** Substitute the controller parameter values K_p and T_d and calculate the transfer function of the closed loop system, $G_{cl1}(s)$. Estimate the resulting Settling Time, $T_{settle(\pm 2\%)}$.

3) **(6 marks)** Assume the same values of the controller parameters K_p and T_d as in item 1), and calculate the transfer function of the closed loop system working under Proportional + Derivative Control, $G_{cl2}(s)$. Compare the two closed loop transfer functions, $G_{cl1}(s)$ and $G_{cl2}(s)$ – what is the difference between them, and how will it affect each of the three response specifications: Percent Overshoot, Steady State Error, Rise Time and Settling Time.

Question 8

Second Order Dominant Poles Model in s-Domain and in Frequency Domain (Open and Closed Loop), Step Response Specifications.

Consider a certain closed loop control system under Proportional Control, as shown in Figure Q8.1. Open loop frequency response plots of the system ($K_p = 1$) are shown in Figure Q8.2 and closed loop frequency response plots of the system ($K_p = 1$) are shown in Figure Q8.3.

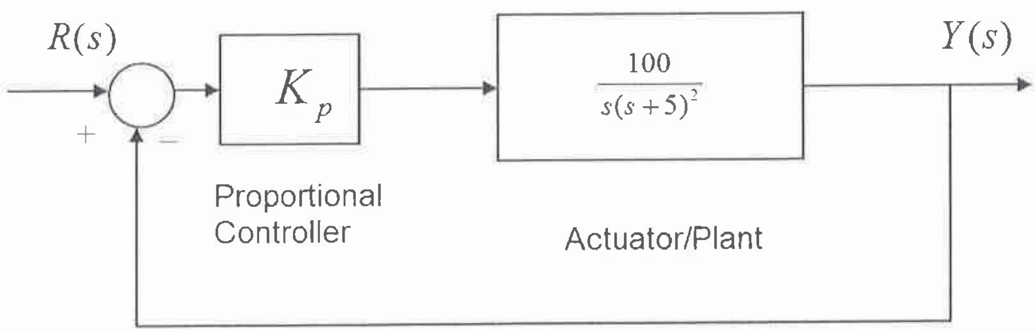


Figure Q8.1

The closed loop transfer function of the system ($K_p = 1$) has been calculated, and its three poles are factorized as follows:

$$p_1 = -0.7791 + j3.3524\,,\ p_2 = -0.7791 - j3.3524,\ p_3 = -8.4418.$$

1) **(5 marks)** Determine if a second order dominant poles model applies to the closed loop transfer function and if so, derive the model transfer function, $G_{m1}(s)$;

2) **(5 marks)** Assume the second order dominant poles model for the closed loop system, based on the information provided in the open loop frequency response plots (Figure Q8.2), and derive the model transfer function, $G_{m2}(s)$;

3) **(5 marks)** Assume the second order dominant poles model for the closed loop system, based on the information provided in the closed loop frequency response (magnitude only) plot (Figure Q8.3), and derive the model transfer function, $G_{m3}(s)$;

4) **(5 marks)** How do the three models compare? Use the one you consider the most accurate to estimate the following closed loop step response specifications: $e_{ss(step\%)}$, $T_{rise(0-100\%)}$ and PO.

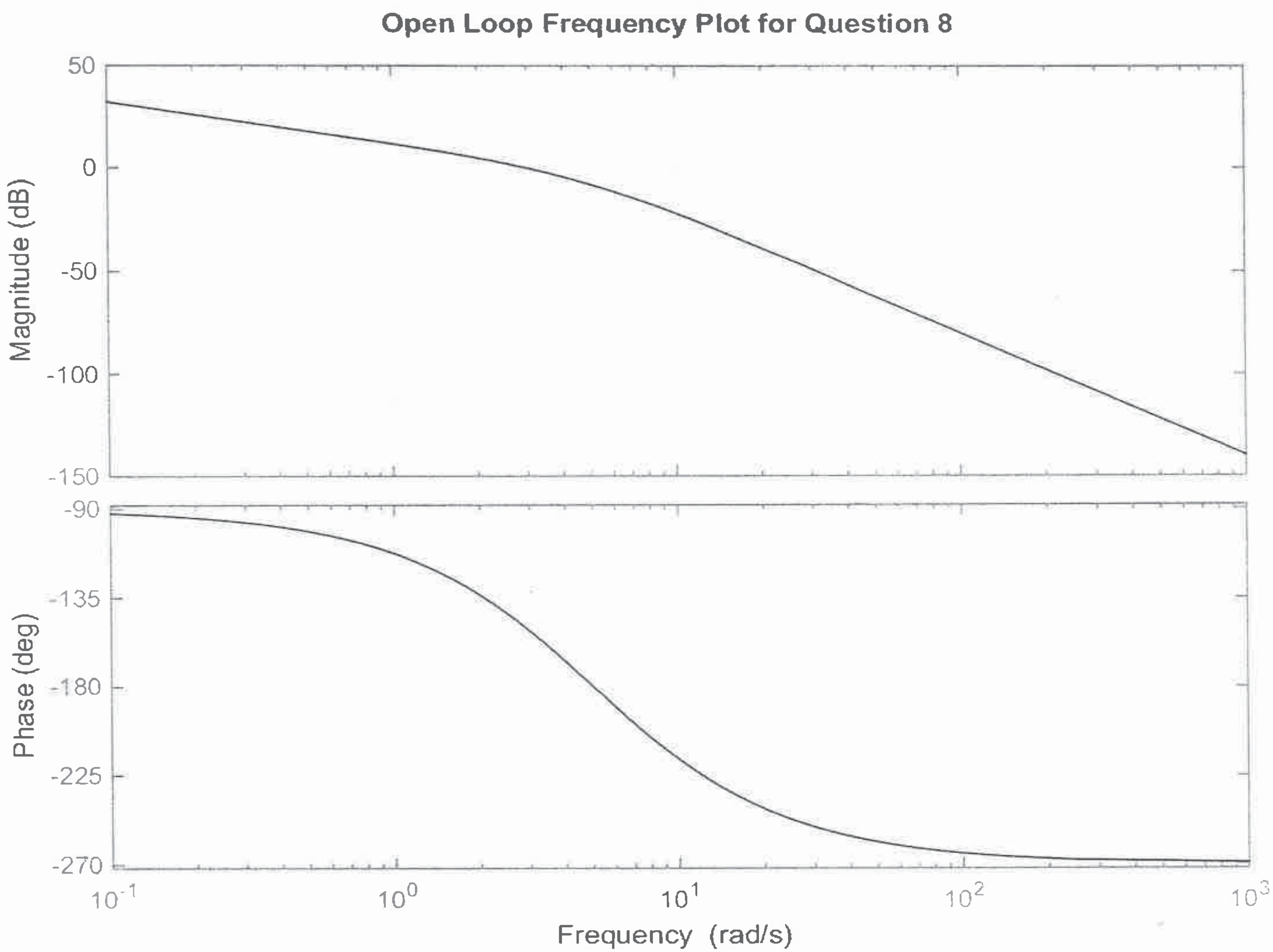


Figure Q8.2 - Open Loop Frequency Response (Magnitude and Phase) Plots for Question 8

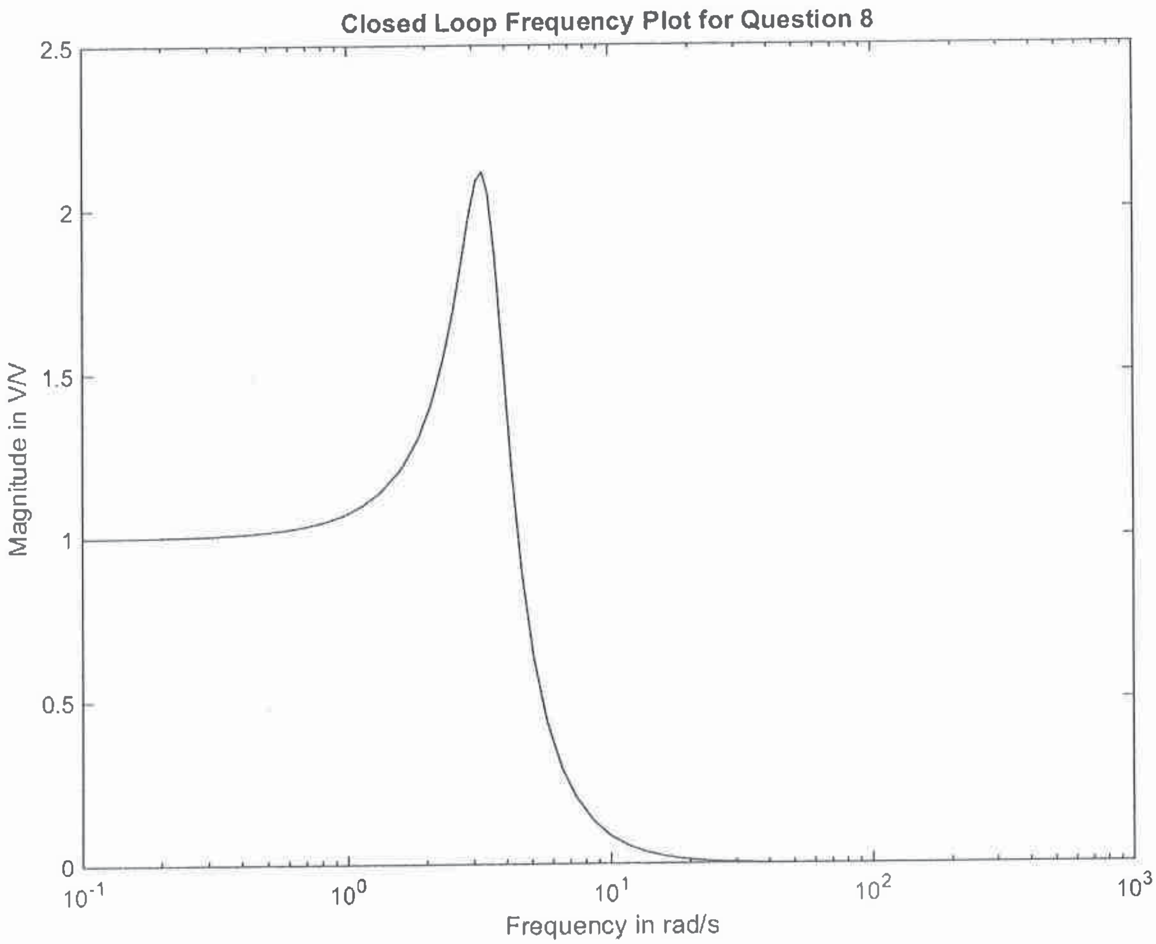


Figure Q8.3 – Closed Loop Frequency Response (Magnitude) Plot for Q8