# National Exams December 2014

# 07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

**NOTES:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

**Question 1:**

Accurately sketch the root locus for the system shown in Figure 1, and give the range of $0 < K < \infty$ for which the system is stable.

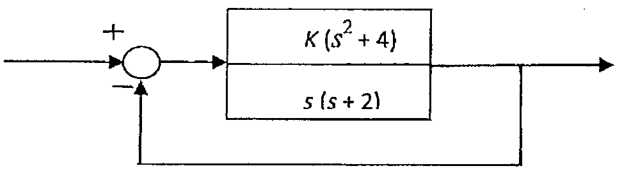



Figure 1

**Question 2:**

Sketch the Bode diagrams for the given transfer functions.

(a) $G(s) = \dfrac{1}{s^2 (s + 1)}$

(b) $G(s) = \dfrac{s}{(s + 1)(s + 1)}$

**Question 3:**

For the system of Figure 2, the input $r(t) = 3 \cos 0.4t$ is applied at $t = 0$.

(a) Find the steady-state system response.

(b) Find the range of time $t$ for which the system is in steady state.

(c) Find the steady-state response for the input $r(t) = 3 \cos 4t$.

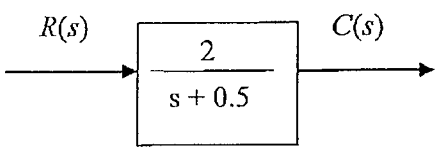



Figure 2

**Question 4:**

Use the Routh-Hurwitz criterion to determine the number of roots in the left half-plane, the right-half plane, and on the imaginary axis for the given characteristic equations.

(a) $s^4 + 2s^2 + 1 = 0$

(b) $s^4 + s^3 + 5s^2 + 5s + 2 = 0$

(c) $s^4 + 2s^3 + 3s^2 + 2s + 5 = 0$

**Question 5:**

Consider the system of Figure 3. Assume that all initial conditions are zero.

(a) If the disturbance $D(s)$ is zero and $R(s)$ is not zero, find the transient-response term in the system output as a function of the gain $K$.
(b) If the disturbance $D(s)$ is not zero and $R(s)$ is zero, find the transient-response term in the system output as a function of the gain $K$.
(c) Find the value of $K$ that will give the closed-loop system a time constant of 1 s.

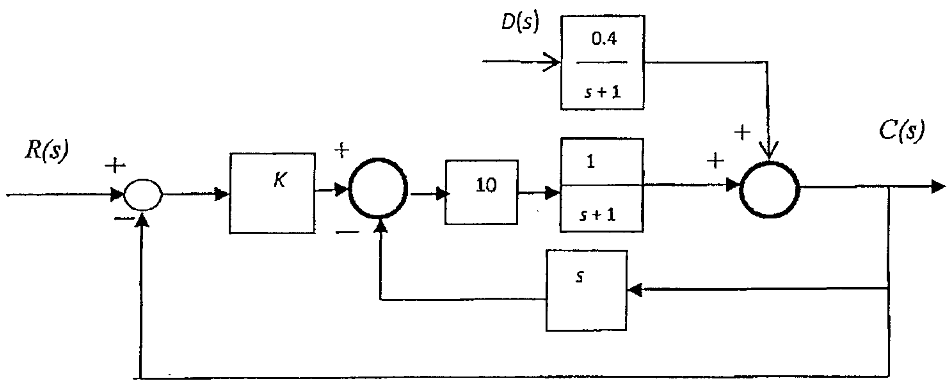



Figure 3

**Question 6:**

In the motor position servo of Fig. 4, let $G(s) = 1/[s\ (s + 1)]$ represent the motor and load and $G_c = K$ the controller/amplifier. Calculate the steady-state errors of the system for unit step and unit ramp signals applied in turn to reference input $R$.

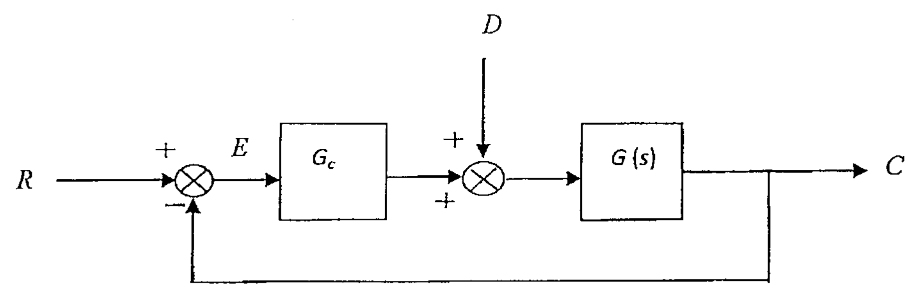
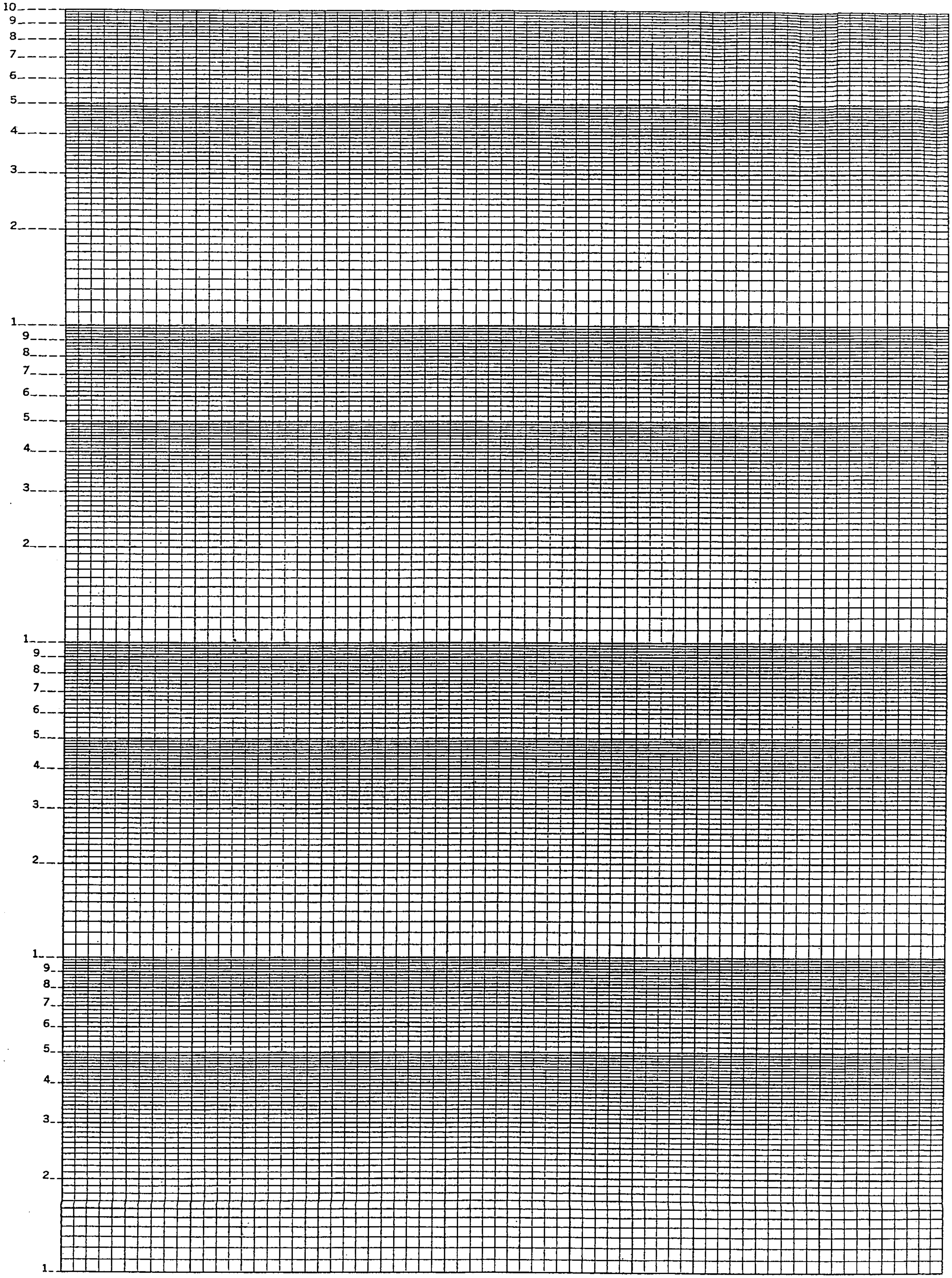



Figure 4

## Laplace Transform Table

| Laplace Transform $F(s)$ | Time function $f(t)$ |
|---|---|
| $1$ | Unit-impulse function $\delta(t)$ |
| $\frac{1}{s}$ | Unit-step function $u_s(t)$ |
| $\frac{1}{s^2}$ | Unit-ramp function $t$ |
| $\frac{n!}{s^{n+1}}$ | $t^n$ ($n$ = positive integer) |
| $\frac{1}{s+\alpha}$ | $e^{-\alpha t}$ |
| $\frac{1}{(s+\alpha)^2}$ | $te^{-\alpha t}$ |
| $\frac{n!}{(s+\alpha)^{n+1}}$ | $t^n e^{-\alpha t}$ ($n$ = positive integer) |
| $\frac{1}{(s+\alpha)(s+\beta)}$ | $\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$ |
| $\frac{s}{(s+\alpha)(s+\beta)}$ | $\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$ |
| $\frac{1}{s(s+\alpha)}$ | $\frac{1}{\alpha}(1-e^{-\alpha t})$ |
| $\frac{1}{s(s+\alpha)^2}$ | $\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha te^{-\alpha t})$ |
| $\frac{1}{s^2(s+\alpha)}$ | $\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$ |
| $\frac{1}{s^2(s+\alpha)^2}$ | $\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$ |

| Laplace Transform $F(s)$ | Time Function $f(t)$ |
|---|---|
| $\dfrac{s}{(s+\alpha)^2}$ | $(1-\alpha t)e^{-\alpha t}$ |
| $\dfrac{\omega_n^2}{s^2+\omega_n^2}$ | $\sin \omega_n t$ |
| $\dfrac{s}{s^2+\omega_n^2}$ | $\cos \omega_n t$ |
| $\dfrac{\omega_n^2}{s(s^2+\omega_n^2)}$ | $1-\cos \omega_n t$ |
| $\dfrac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$ | $\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$ |
| $\dfrac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$ | $\dfrac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\dfrac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$ |
| $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ | $\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$ |
| $\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ | $1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$ |
| $\dfrac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ | $\dfrac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$ |
| $\dfrac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$ | $\omega_n\sqrt{\dfrac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\dfrac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$ |
| $\dfrac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$ | $t-\dfrac{2\zeta}{\omega_n}+\dfrac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$ |

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