National Exams May 2019

16-CHEM-B1, TRANSPORT PHENOMENA

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

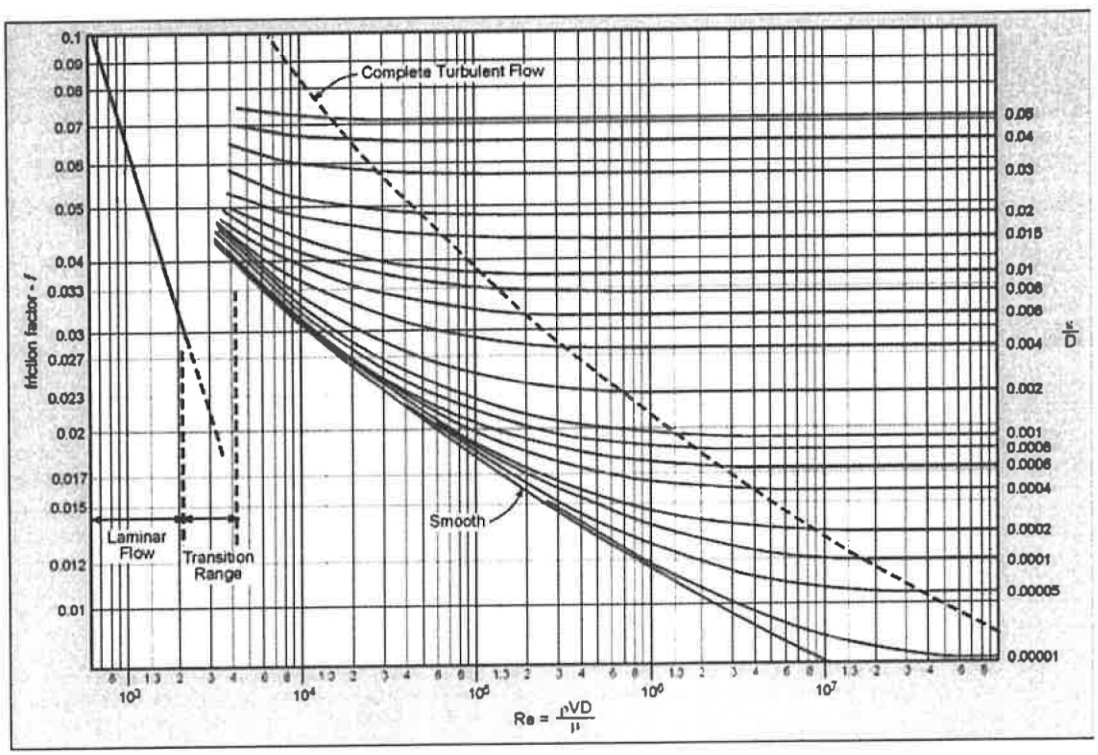
SECTION A: Fluid Mechanics

A1. Benzene (density = 0.899 g/cm^3, viscosity = 8 x 10^{-4} N.s/m^2) flows steadily through a 150-meter long horizontal pipe of 5.5 cm inside diameter at a flow rate of 15 liters/min. The equivalent roughness for the pipe is 8.5 x 10^{-4} ft.

a) [13 points] Calculate the pressure drop.

b) [10 points] What is the pressure drop if the same amount of kerosene (density = 0.82 g/cm^3, viscosity = 2.5 x 10^{-3} N.s/m^2) is flowing through the same pipe?

c) [2 points] Explain the difference in pressure drop between (a) and (b).



Moody friction factor (f) vs. Reynolds number (Re) for pipes

A2. A polymer melt follows the power law for variation of shear stress (τ_{yx}) with shear strain rate (dV_x/dY) given by the following equation:

$$\tau_{yx} = -\eta_o [dV_x/dY]^n$$

where $\eta_o \rightarrow$ zero-shear viscosity

$V_x \rightarrow$ local velocity in the x-direction

$n \rightarrow$ Power law index

Derive an equation for the velocity profile (V_x) and volumetric flow rate (Q) for flow between two parallel plates. The direction between the parallel plates is the Y-direction.

SECTION B: Heat Transfer

B1. A 2.5 mm thick, 2.5 m long square steel plate is removed from an oven at 430 K to an atmosphere at 295 K. Calculate the initial heat loss (W) for the following conditions:

- **a) [15 points]** The plate is hung horizontally.
- **b) [10 points]** The plate is hung vertically.

B2. Consider fluid flow in a circular pipe of radius R. The velocity distribution of the fluid is slug flow (v_z is uniform) and heat flux (q_0) is uniform. The heat transfer coefficient (h) is defined as $q_0/(T_R - T_m)$, where T_R is the temperature of the fluid at the tube wall and T_m is the mean temperature of the fluid. Show that the value for Nusselt number for fully developed flow is 8.

SECTION C: Mass Transfer

C1. Consider the slow reversible decomposition of a metal oxide (MO) to the metal (M) and oxygen (O) given by the equation **MO (s)** $\leftrightarrow$ **M + O**

When melting is done under vacuum, both elements (M and O) are dissolved in the melt. The equilibrium constant (K) for the reaction is given by

$$\mathbf{K = C_M\, C_O = 1 \times 10^{-6}}$$

where $C_M \rightarrow$ concentration of metal in moles/cm^3 of melt
$C_O \rightarrow$ concentration of oxygen in moles/cm^3 of melt

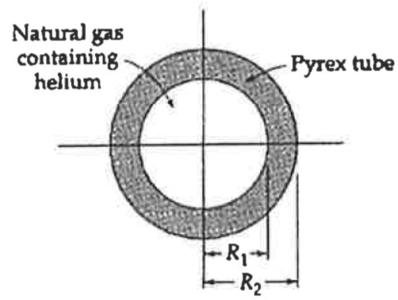
We are interested in flow of melt parallel to a 3-meter long flat plate of the metal oxide. The concentration of metal is zero before the leading edge of the plate and the bulk flow velocity of melt far away from the plate is 3 m/s. The concentration of the metal and oxygen in the melt varies along the length of the flat plate but they are of same value at any particular point on the plate.

a) **[15 points]** Calculate the average mass transfer coefficient for the metal dissolving in the melt.

b) **[10 points]** Calculate the average flux of metal oxide dissolving in the melt.

DATA FOR MELT: Density = 8 g/cm^3
Viscosity = 1.24 x 10^{-3} N.s/m^2
Thermal conductivity = 50 W/m.K
Specific heat capacity = 840 J/Kg.K
Diffusivity of oxygen/metal = 5 x 10^{-9} m^2/s

C2. Pyrex glass is almost impermeable to all gases except helium. A method for separating helium from natural gas could be based on the relative diffusion rates through Pyrex. Suppose a natural gas mixture is contained in a Pyrex tube as shown in the figure below:



Obtain an expression for the rate at which helium will "leak" out of the tube in terms of diffusivity of helium through Pyrex, interfacial concentrations of helium in the Pyrex, and dimensions of the tube.

Table 1: Properties of Dry Air at One Atmosphere

Density and specific heat are from Hilsenrath (1955); thermal conductivity and viscosity are from Touloukian (1970); other values are calculated.

T $[K]$	ρ $\left[\frac{kg}{m^3}\right]$	μ $\left[10^{-6}\frac{N\cdot s}{m^2}\right]$	κ $\left[10^{-3}\frac{W}{m\cdot K}\right]$	C_p $\left[\frac{J}{kg\cdot K}\right]$	ρ/μ $\left[10^3\frac{s}{m^2}\right]$	$g\beta/(\nu\alpha)$ $\left[10^6\frac{1}{m^3\cdot K}\right]$	α $\left[10^{-6}\frac{m^2}{s}\right]$
200	1.7690	13.36	18.10	1006.4	132.4	638.6	10.17
210	1.6842	13.92	18.95	1006.1	121.0	505.2	11.18
220	1.6071	14.47	19.80	1005.7	111.1	404.2	12.25
230	1.5368	15.01	20.63	1005.6	102.4	327.0	13.35
240	1.4728	15.54	21.45	1005.5	94.8	267.3	14.49
250	1.4133	16.06	22.26	1005.4	88.0	220.4	15.67
260	1.3587	16.57	23.05	1005.5	82.0	183.3	16.87
270	1.3082	17.07	23.84	1005.5	76.6	153.6	18.12
280	1.2614	17.57	24.61	1005.7	71.8	129.6	19.40
290	1.2177	18.05	25.38	1006.0	67.5	110.1	20.72
300	1.1769	18.53	26.14	1006.3	63.5	94.1	22.07
310	1.1389	19.00	26.87	1006.8	59.9	80.9	23.43
320	1.1032	19.46	27.59	1007.3	56.7	70.0	24.83
330	1.0697	19.92	28.30	1007.9	53.7	60.8	26.25
340	1.0382	20.37	29.00	1008.5	51.0	53.1	27.70
350	1.0086	20.81	29.70	1009.2	48.5	46.5	29.18
360	0.9805	21.25	30.39	1010.0	46.1	41.0	30.69
370	0.9539	21.68	31.07	1010.9	44.0	36.2	32.22
380	0.9288	22.11	31.73	1012.0	42.0	32.1	33.76
390	0.9050	22.52	32.39	1013.0	40.2	28.6	35.33
400	0.8822	22.94	33.05	1014.2	38.5	25.5	36.94

APPENDIX

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t}+(\nabla \cdot \rho \vec{u})=0$$	(1.1)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u_x)+\frac{\partial}{\partial y}(\rho u_y)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1a)
Cylindrical coordinates (*r*, θ, *z*) $$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1b)
Spherical coordinates (*r*, θ, ϕ) $$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi)=0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{g}+\nu(\nabla^2\vec{u})$$	(A2)
Rectangular coordinates (*x*, *y*, *z*)		
x-component	$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+g_x+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g_y+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (*r*, *θ*, *z*)

r-component (A2d)

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \tag{A2d}$$

θ-component (A2e)

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}$$
$$=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \tag{A2e}$$

z-component (A2f)

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \tag{A2f}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

r-component (A2g)

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r$$
$$+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial\phi^2}\right] \tag{A2g}$$

θ-component (A2h)

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta$$
$$+\nu\left[\begin{aligned}&\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial\phi^2}\\&+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial\phi}\end{aligned}\right] \tag{A2h}$$

ϕ-component (A2i)

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}$$
$$+g_\varphi+\nu\left[\begin{aligned}&\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial\phi^2}\\&+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial\phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial\phi}\end{aligned}\right] \tag{A2i}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=[\nabla\cdot k\nabla T]+\dot{T}_G \tag{A3}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \tag{A3a}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \tag{A3b}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\begin{aligned}&\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=\\&\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G\end{aligned} \tag{A3c}$$

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G} \tag{4.}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G} \tag{4a}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G} \tag{4b}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G} \tag{4c}$$

Table A.5: The continuity equation for species A

$\frac{\partial C_A}{\partial t}+(\vec{u}\cdot\nabla)C_A=D_A\nabla^2 C_A+\dot{R}_{A,G}$	(5)
Rectangular coordinates (x, y, z) $\frac{\partial C_A}{\partial t}+u_x\frac{\partial C_A}{\partial x}+u_y\frac{\partial C_A}{\partial y}+u_z\frac{\partial C_A}{\partial z}=\frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right)+\frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right)+\dot{R}_{A,G}$	(5a)
Cylindrical coordinates (r, θ, z) $\frac{\partial C_A}{\partial t}+u_r\frac{\partial C_A}{\partial r}+\frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta}+u_z\frac{\partial C_A}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right)+\dot{R}_{A,G}$	(5b)
Spherical coordinates (r, θ, ϕ) $\frac{\partial C_A}{\partial t}+u_r\frac{\partial C_A}{\partial r}+\frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D\frac{\partial C_A}{\partial r}\right)$ $+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right)+\dot{R}_{A,G}$	(5c)