National Exams May 2002
98-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Any non-communicating calculator is permitted.
   This is an Open Book Exam.
   However candidates are allowed to bring only ONE aid sheet 8.5" x 11" hand-written on bothSides containing notes & formulae and one textbook of their choice.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value. Each question is worth 20 marks.

1. (a) Solve the initial value problem

$$y'' + 9y = 6\cos(3t), \qquad y(0) = 0,\ y'(0) = 0.$$

   (b) Find the general solution of the differential equation

$$y' + 2xy = e^{-x^2}\sec(2x)$$

2. Find the general solution of the differential equation

$$x^2y'' - 4xy' + 6y = 3x^4.$$

3. Find the maximum and minimum values of $f(x,y,z) = x + y - z$ over the sphere $x^2 + y^2 + z^2 = 1$.

4. Find the centre of mass of the solid bounded by the two paraboloids $z = x^2 + y^2$ and $z = 4 - 2x^2 - 2y^2$ whose density is $\rho(x,y,z) = 3z$.

5. Let $A = \begin{pmatrix} 4 & 5 \\ 3 & 2 \end{pmatrix}$.

   (a) Without using a calculator, find the eigenvalues and eigenvectors of A.

   (b) Solve the initial value problem

$$\begin{aligned} x' &= 4x + 5y, \qquad x(0) = 1, \\ y' &= 3x + 2y, \qquad y(0) = 0. \end{aligned}$$

6. Find the equation of the plane tangent to the surface defined implicitly by $xy^2z^3 = 1 + x$ at the point $(8, 3, 1/2)$.

7. Evaluate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dA$, where $\mathbf{F}(x,y,z) = \mathbf{i} - y\mathbf{j} + 3z\mathbf{k}$ and S is the surface of the region bounded by the cone $z = 4 - x^2 - y^2$ and the plane $z = 0$.

8. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 1+y$, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2x\mathbf{j} + 2x\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v}\cdot d\mathbf{r}$.