National Exams May 2002

98-BS-5 Advanced Mathematics
3 Hours duration

NOTES:

1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **TWO** aid sheets written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

20 marks

2. 20 marks

3. 20 marks

4. 20 marks

5. A 10 marks, B 10 marks

6. A 10 marks, B 10 marks

7. 20 marks

8. 20 marks

May 2002 Advanced Mathematics

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Instructions: Please refer to the front page of this examination for detailed instructions.

1. Solve the following differential equation using Laplace Transform.

$$L\frac{di}{dt}+\frac{1}{C}\int_0^t id\tau = v(t)$$
$$L=1$$
$$C=1$$
$$v(t)=\begin{cases}1-e^{-t} & \text{if } 0<t<1\\ 0 & \text{if } t>1\end{cases}$$
$$i\big|_{t=0}=0$$

2. Solve the following initial problem using the method of power series.

$$\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$$
$$y\big|_{x=0}=1$$
$$\left.\frac{dy}{dx}\right|_{x=0}=1$$

3. Find the Fourier cosine integral of $f(x)$

$$f(x)=\begin{cases}x^2 & \text{if } 0<x<2\\ 0 & \text{if } x>2\end{cases}$$

May 2002 Advanced Mathematics

4. Find the Fourier series of the following function $f(x)$, which is assumed to have the period p = 2.

$$f(x)=\begin{cases} x & \text{if } -\frac{1}{2}<x<\frac{1}{2} \\ 1-x & \text{if } \frac{1}{2}<x<\frac{3}{2} \end{cases}$$

5. A. Compute $\int_0^1 f(x)dx = \int_0^1 e^{x^2}dx$ by using the composite Trapezoid rule with six uniform points.

B. What numerical value will the error not exceed, i.e. what is the maximum numerical error?

Hint: If f'' exists and is continuous on the interval [a, b], and if the composite trapezoid rule with uniform spacing h is used to estimate the integral $\int_a^b f(x)dx$, then the error term is $-\frac{1}{12}(b-a)h^2 f''(\zeta)$. Where $a \le \zeta \le b$.

x_i	$f(x_i)=e^{x_i^2}$
0	1.00000
0.2	1.04081
0.4	1.17351
0.6	1.43333
0.8	1.89648
1.0	2.71828

6. Solve the following initial-value problem on the interval [0, 0.6] by using the two methods.
   A. Eller's method (the Taylor series method of order 1) with three steps.
   B. The Taylor series method of order 2 with three steps.

$$\begin{cases} \dfrac{dx}{dt} + tx = t^3 \\ x\big|_{t=0} = 1 \end{cases}$$

7. Solve the following linear system AX = B by finding the inverse of the coefficient matrix A.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad B = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$$

8. Find the eight equations for determining the eight coefficients a_{ij} so that S(x) is a natural cubic spline, where

$$S(x) = \begin{cases} a_{10} + a_{11}x + a_{12}x^2 + a_{13}x^3 & -1 \le x \le 0 \\ a_{20} + a_{21}x + a_{22}x^2 + a_{23}x^3 & 0 \le x \le 1 \end{cases}$$

with interpolation conditions S(-1)=1, S(0)=2, and S(1)=0.