National Exams May 2003

98-BS-5 Advanced Mathematics

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheets written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. 20 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. A 5 marks, B 5 marks, C 10 marks
8. 20 marks

98-BS-5
Advanced Mathematics

Instructions: Please refer to the front page of this examination for detailed instructions.

1. Solve the following differential equation using Laplace Transform.

$$\frac{d^2y}{dt^2} + 4y = r(t)$$

$$r(t) = \begin{cases} 3\sin t & 0 < t < \pi \\ -3\sin t & t > \pi \end{cases}$$

$$y(0) = 0, \qquad \left.\frac{dy}{dt}\right|_{t=0} = -1$$

2. Apply the power series method to solve the following differential equation.

$$(1-x^2)\frac{d^2y}{dx^2} - 2xy\frac{dy}{dx} + 2y = 0$$

$$y|_{x=0} = 1$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 0$$

3. Find the Fourier cosine integral of $f(x)$.

$$f(x) = \begin{cases} x/2 & 0 < x < a \\ 0 & x > a \end{cases}$$

4. Find the Fourier series of the periodic function $f(x)$, which has a period of 1 ($p = 1$).

$$f(x) = \pi \sin \pi x \qquad\qquad 0 < x < 1$$

5. Compute $\int_0^4 2^x dx$ by the Romberg algorithm using $n = 2$. What is the analytical solution?

Hint: To estimate $\int_a^b f(x)dx$, the Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$.

$$R(0,0) = \frac{1}{2}(b-a)[f(a)+f(b)]$$

$$R(n,0) = \frac{1}{2}R(n-1,0) + h\sum_{k=1}^{2^{n-1}} f[a+(2k-1)h]$$

$$\text{Where} \quad h = \frac{b-a}{2^n}$$

$$R(n,m) = R(n,m-1) + \frac{1}{4^m - 1}[R(n,m-1) - R(n-1,m-1)]$$

6. Calculate an approximate value for $x\big|_{t=0.1}$, using one step of the Taylor series method of order 3, for the following differential equation.

$$\begin{cases} \dfrac{d^2x}{dt^2} = x^3e^t + \dfrac{dx}{dt} \\ x\big|_{t=0} = 1 \\ \dfrac{dx}{dt}\bigg|_{t=0} = 2 \end{cases}$$

7. Solve the following system of equations assuming that the number of significant figures carried in the computation is 5.

$$\begin{cases} 0.00010x_1 + 1.0000x_2 = 0.66670 & (1) \\ 1.0000x_1 + 1.0000x_2 = 1.0000 & (2) \end{cases}$$

A. By naive Gaussian elimination method (the order in which the equations are used as pivot equations is the natural order (1,2,…., n)).
B. By Gaussian elimination method with scaled partial pivoting.
C. Explain why the naive Gaussian elimination method fails to provide a correct solution and how to choose the pivot equation in the Gaussian elimination method with scaled partial pivoting.

8. Find an interpolating polynomial (Newton Form is preferred) for the following given data and determine the value of $f(2.2)$.

x	1	2	2.5	3
$f(x)$	-1	$-\frac{1}{3}$	$\frac{3}{32}$	$\frac{4}{3}$