

National Exams May 2003

98-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

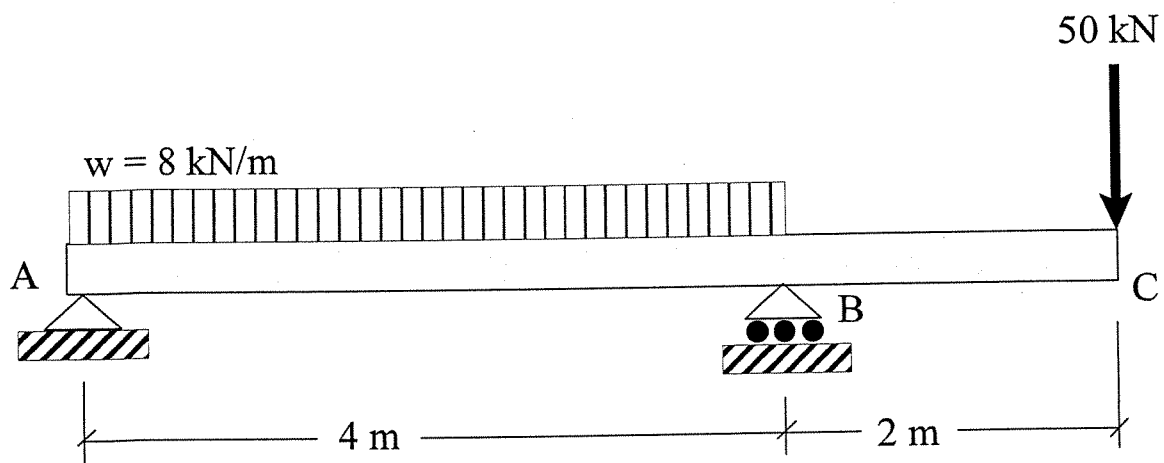
- ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. Example problems and solutions to problems are not allowed!
3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
 4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

Question 1: A simply supported beam with a 2 m overhang supports a uniformly distributed load and an applied load at the end of the overhang as shown.

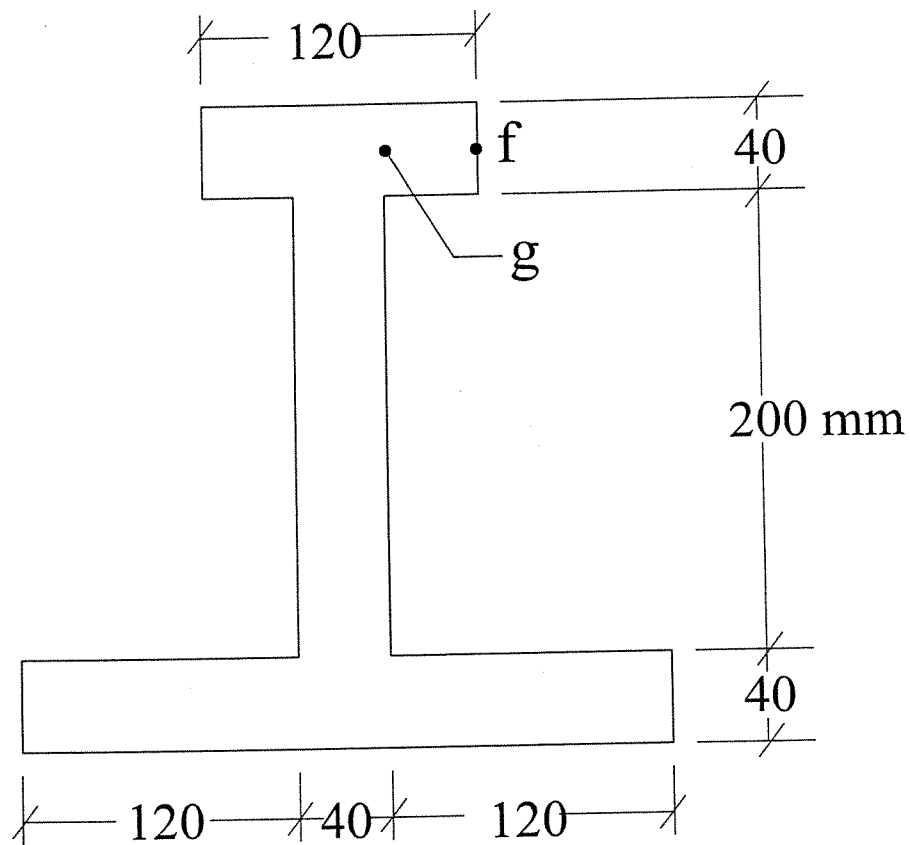
[20 marks] For this beam, determine the shear and moment throughout the beam as functions of x , and then sketch the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points.

Note that **no credit** will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.



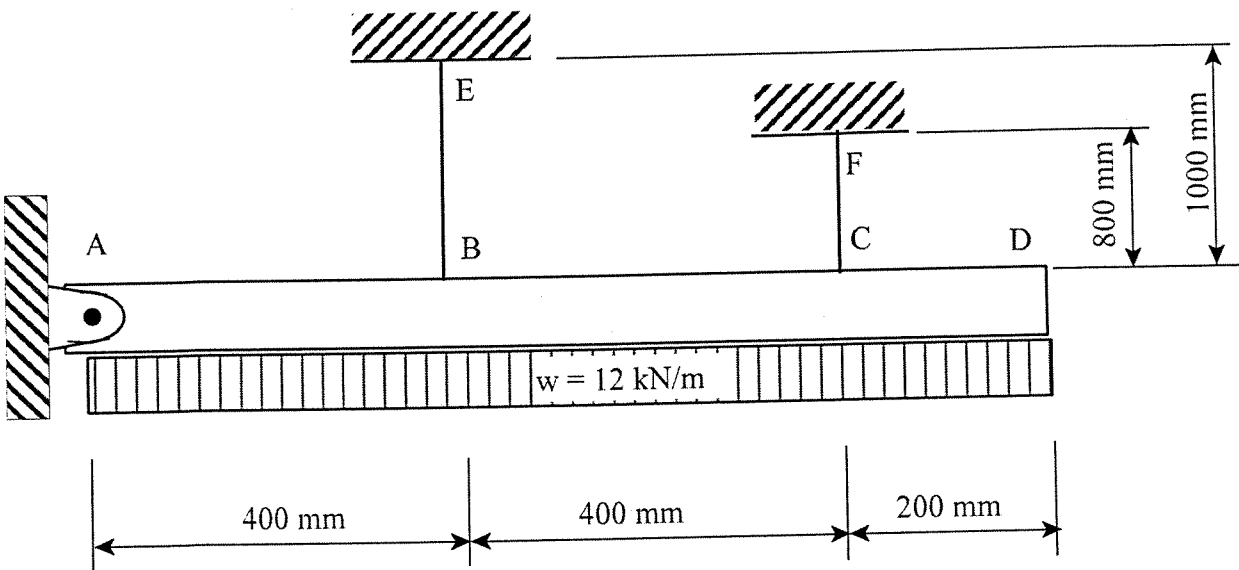
Question 2: The simply supported beam from Question 1 has a 4 m span with a 2 m overhang, and supports a uniformly distributed load of 8 kN/m in addition to a concentrated load of 50 kN at the end of the overhang. Assume an elastic modulus of $E = 200 \text{ GPa}$ and yield stress of $\sigma_y = 350 \text{ MPa}$.

- [18 marks] (a) determine the maximum normal stress and maximum shear stress in the beam.
- [2 marks] (b) calculate the maximum shear stress at the points f and g indicated on the cross-section.



Question 3: A distributed load of $w = 12 \text{ kN/m}$ is placed on a rigid bar that is supported by a pin at A and two steel wires at B and C. Both wires have a diameter of 5 mm but different lengths as shown. The yield stress for the wires is $\sigma_y = 400 \text{ MPa}$ and the elastic modulus $E = 200 \text{ GPa}$.

- [12 marks] (i) determine the force in each of the two wires,
- [4 marks] (ii) find the corresponding displacement at the end of the rigid bar (point D), and
- [4 marks] (iii) compute the shear stress in the bolt at the pinned connection A, assuming that this bolt has a diameter of 25 mm and is in double shear.

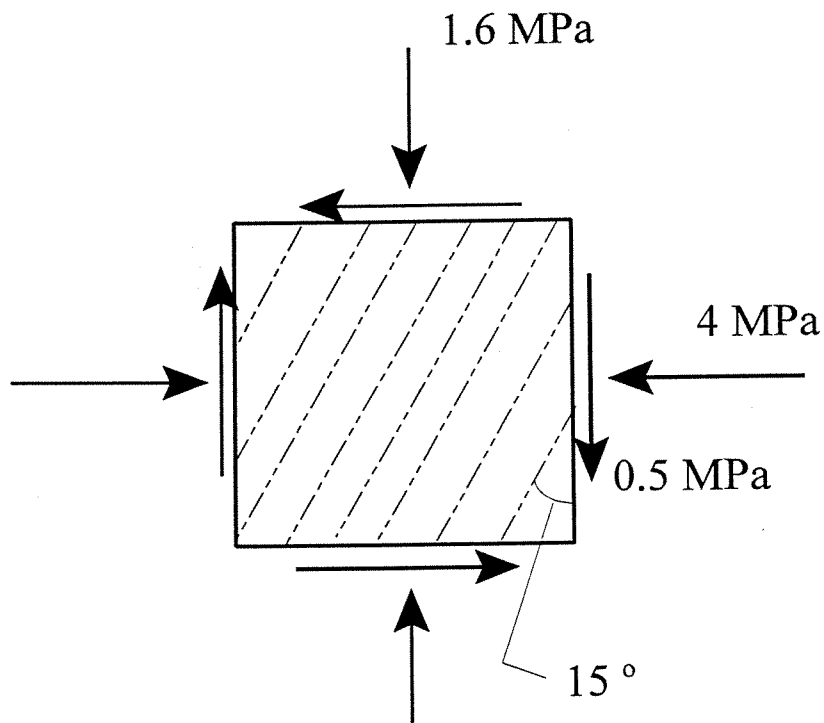


Question 4: The grain of a wooden member forms an angle of 15° with the vertical, and is subjected to the state of stress as shown below.

[18 marks] Use the Mohr's circle solution (*not* the transformation equations) to determine the normal stress perpendicular to the grain and the in-plane shear stresses acting along the grain. Show your answer on an inclined element.

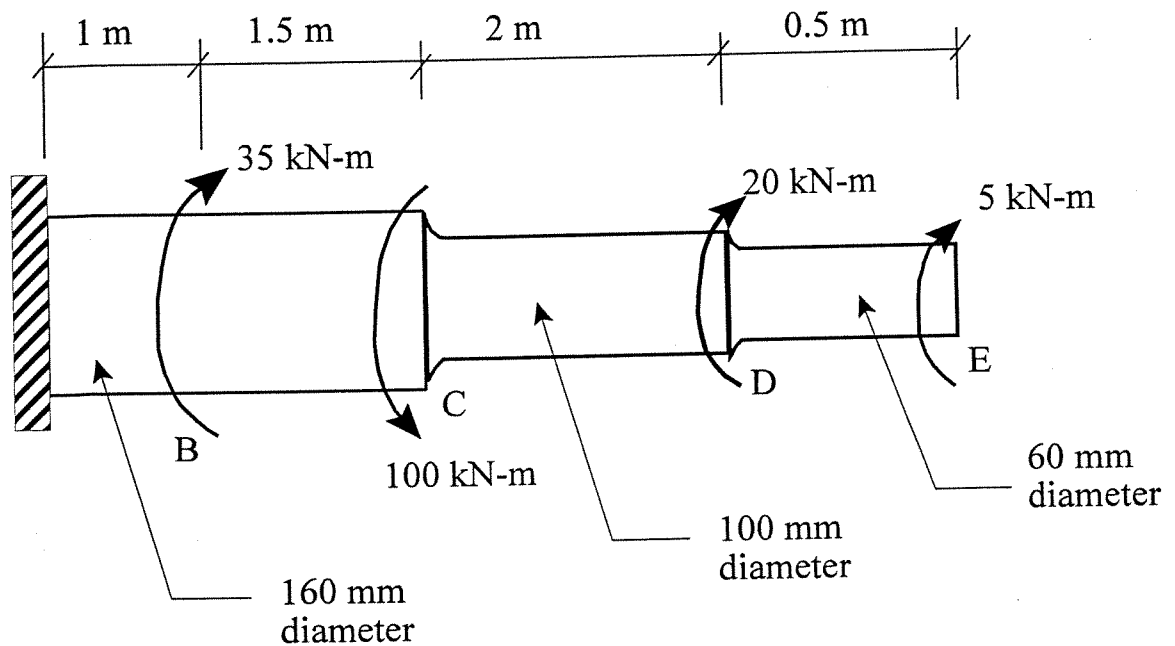
[2 marks] Why is it necessary to calculate the stress acting along the grain?

Note that credit will **only** be given for a **solution using Mohr's circle**. Stress transformation equations can only be used to check your answer.



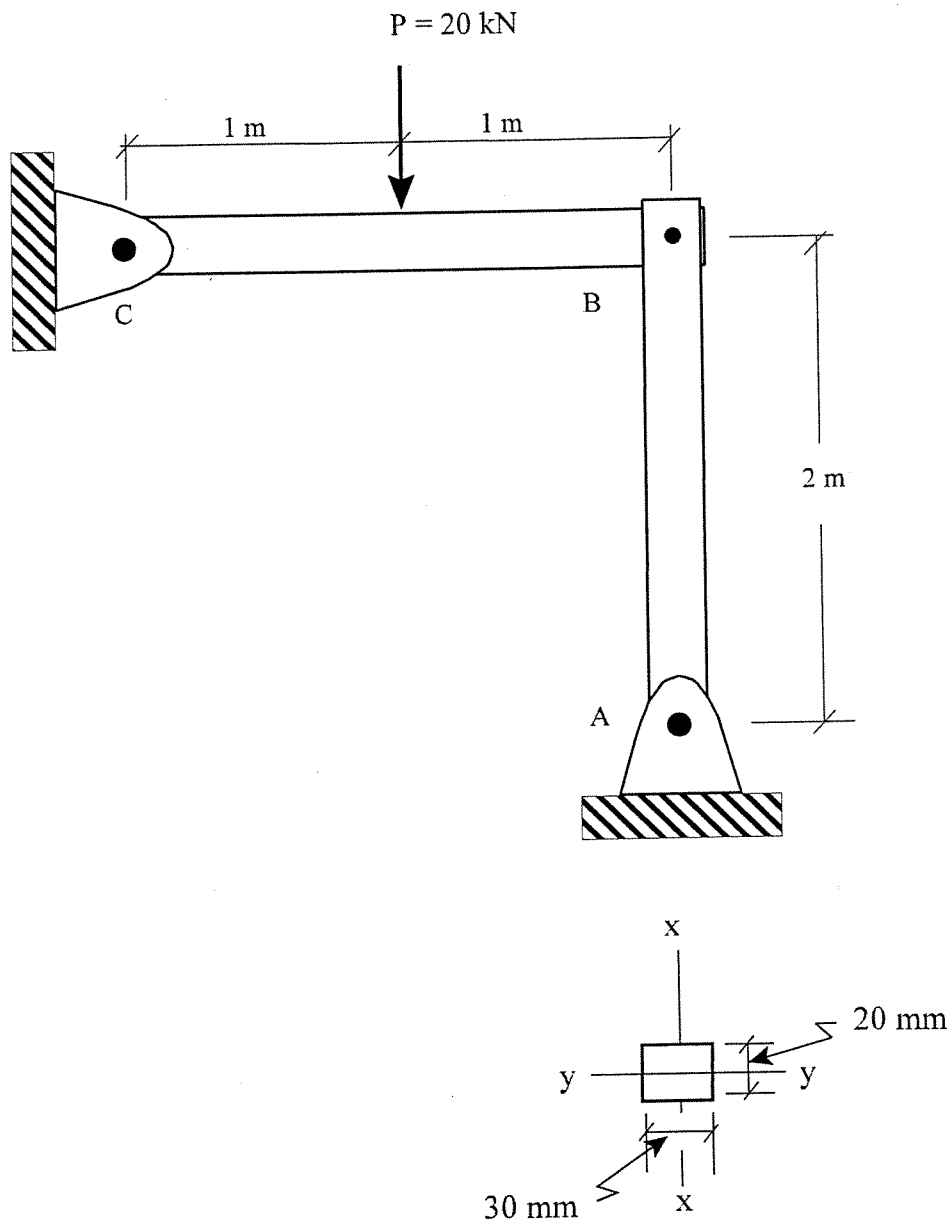
Question 5: A solid stepped shaft with a shear modulus of $G = 80 \text{ GPa}$ and yield stress of $\tau_y = 150 \text{ MPa}$ has the dimensions shown. The applied torques are as indicated in the figure.

- [12 marks] (i) determine the maximum shear stress in the shaft, and sketch the corresponding variation of shear stress along the shaft radius.
- [6 marks] (ii) find the angle of twist at the end of the shaft and give your answer in degrees.
- [2 marks] (iii) indicate whether the shaft will fail, assuming that the step in the shaft at C and D is constructed with a fillet having a radius sufficient to reduce the torsional stress concentration factor to a value of 1.3.



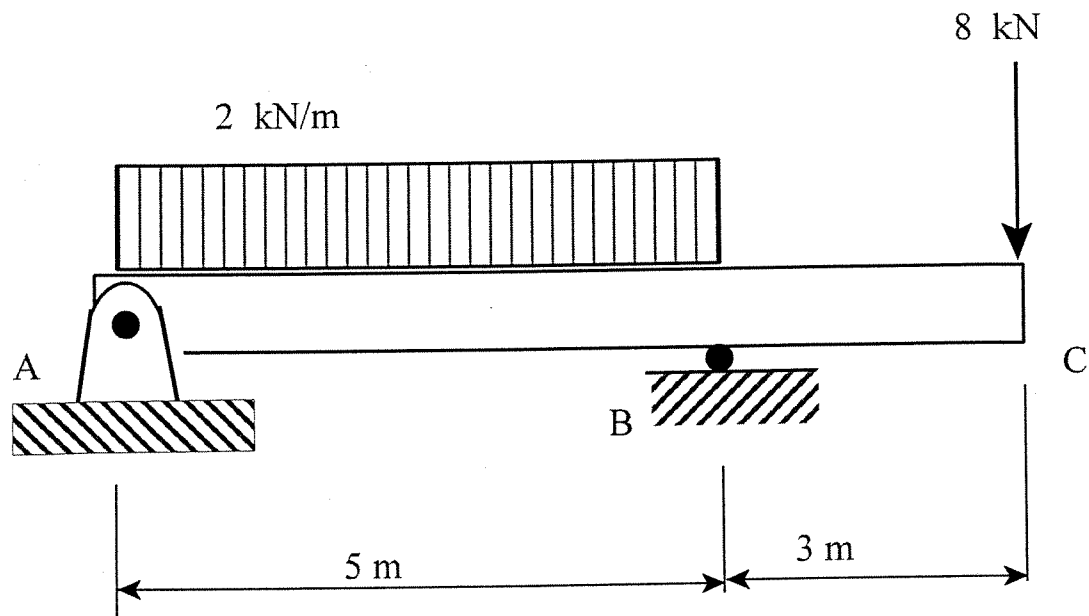
Question 6: Determine if the frame can support a load of $P = 20 \text{ kN}$ if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x-x axis buckling and fixed at its ends for y-y axis buckling. $E = 200 \text{ GPa}$ and $\sigma_y = 360 \text{ MPa}$.

[20 marks]



Question 7 A beam with constant EI is subjected to the loading shown.

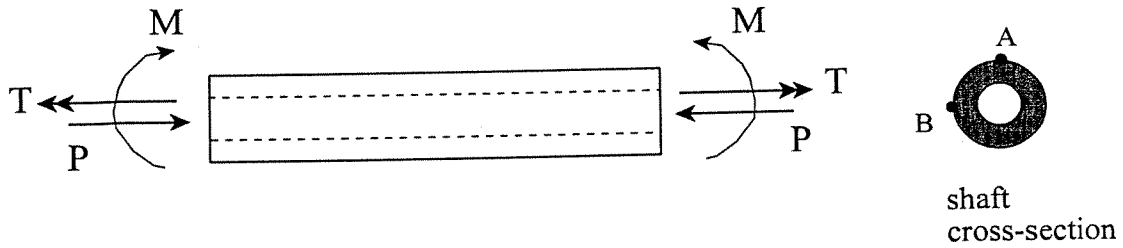
[20 marks] Using the method of integration, determine the slope at A, deflection at C, and sketch the deflected shape of the structure.



Note: since your answer depends on having the correct expression for the bending moment $M(x)$, it is important that you set this problem up correctly by calculating the correct support reaction forces.

Question 8: A generator shaft of hollow circular cross-section (outside diameter of 200 mm and inside diameter of 160 mm) is subjected to the combined action of a torque $T = 25 \text{ kN-m}$, axial compressive load $P = 725 \text{ kN}$, and moment $M = 5 \text{ kN-m}$ as indicated in the figure below.

[20 marks] Determine the stress components on an element of material located at the outside surface of the shaft at points A and B.



Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$