

**National Exams May 2002**

**98-Chem-A6, Process Dynamics and Control**

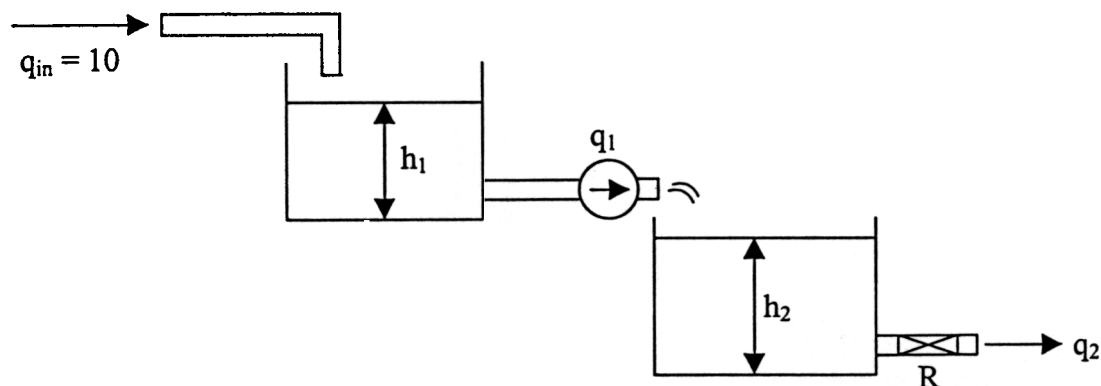
**3 hours duration**

**NOTES:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Any non-communicating calculator is permitted. This is an Open Book exam. You must indicate the type of calculator being used, ie, write the name and model designation of the calculator on the first inside left hand sheet of the exam work book.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Note 1: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

**Problem #1** (20% total)



Two tanks are connected in series in a noninteracting fashion as shown in the figure.

Assume:  $\rho = 1$   $A = 1$  (A-cross-section of each tank)

$$q_2 = \frac{1}{R} \sqrt{\frac{\Delta P}{\rho g}} \text{ and } q_1 \text{ is determined by a pump.}$$

$q_{in} = 10$  and remains constant. The initial level in tank 1 is  $h_1(t=0) = 10$ .  $q_1$  is the manipulated variable. All  $q$ 's are volumetric flow rates.  $R = 2$ .

- (10%) (a) Show the differential equations that describe the behaviour of  $h_1(t)$  and  $h_2(t)$ .
- (10%) (b) Compute transfer functions between  $h_1$  to  $q_{in}$  and  $h_2$  to  $q_{in}$ .

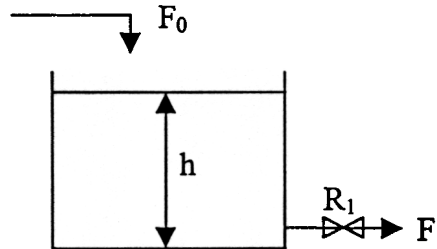
## 98-Chem-A6, Process Dynamics and Control

---

Note 1: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

### Problem #2 (20% total)

For the draining tank shown in the figure



Compute the change in level  $\delta h(t)$  as a function of time for the following two cases:

(10%) (a) a step of one unit in inlet flow  $F_0$

(10%) (b) a unit impulse in inlet flow  $F_0$

The cross-section area of the tank is  $1 \text{ m}^2$ . Initial level =  $1 \text{ m}$

The flow out is given by  $F_1 = R_1 \cdot h$ , where the hydraulic resistance  $R_1 = \frac{1 \text{ m}^2}{\text{min}}$

### Problem #3 (20% total)

A process is described by the following transfer function:

$$G_p = \frac{10(0.5 - s)e^{-10s}}{1 + s}$$

(10%) (a) Design an IMC (Internal Model Controller) for this process. Show your design with a block diagram.

(10%) (b) Assuming a perfect model of the process, compute the closed loop response for a unit step in set point if the desired closed loop time constant is equal to 5.

Note 1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

**Problem #4** (20% total)

For the equation

$$\frac{d^2y}{dt^2} + k \frac{dy}{dt} + 10y = 2x$$

- (10%) (a) Find the transfer function between the input  $x$  to the output  $y$  and put it in the standard gain-time constant form.
- (b) Discuss for which values of  $k$  is the open loop response for a unit step in  $x$  (i) stable, (ii) underdamped, and (iii) overdamped.
- (c) If the response is underdamped, compute expressions as a function of  $k$  for the time constant and the damping coefficient according to the standard form definitions.

**Problem #5** (20% total)

A second order process is given by

$$G_p(s) = \frac{1}{s^2 + 2s + 1}$$

This process is controlled by a proportional-derivative (PD) controller given by:

$$G_c(s) = k_c(1 + s)$$

- (10%) (a) Compute values of the  $k_c$  that will result in closed loop stability. Use Routh-test.
- (10%) (b) If instead of a PD, a proportional only controller  $G_c(s) = 2$  is used, compute the closed loop response as a function of time for a unit step change in set point.

Note 1: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

**Problem #6** (20% total)

A process given by:

$$G_p = \frac{1}{s^2 - s - 2}$$

is controlled by a proportional controller with gain  $k_c$ .

- (10%) (a) Show a qualitative Nyquist plot (show only 2-3 key points and the general shape of the plot for this problem) for  $k_c = 1$ . Assess the closed loop stability based on the Nyquist criterion.
- (10%) (b) Based on the Nyquist criterion, compute a range of  $k_c$  values to obtain closed loop stability.

**Problem #7** (20% total)

The dynamic response of the reactant concentration in a CSTR reactor,  $C_A$ , to a change in inlet concentration,  $C_{A_0}$ , has to be evaluated.

The reactor is operated with constant volume  $V$  and isothermal conditions. The density  $\rho$  is constant.

The reaction rate is:

$$F_A = k_1 C_A^2$$

The mass flowrate is  $F$ .

- (10%) (a) Derive a mathematical model to describe  $C_A(t)$  and compute steady state conditions for concentration.
- (10%) (b) Compute a transfer function  $\delta C_A / \delta C_{A_0}$  (where  $\delta$  indicates deviation variables) when the system is operated around the steady state computed in (a).

## 98-Chem-A6, Process Dynamics and Control

---

Note 1: If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

### Problem #8 (20% total)

A process is described by the following transfer function:

$$G_p = \frac{e^{-2s}}{(s+1)}$$

The process is controlled by a proportional controller with gain  $k_c$ .

- (10%) (a) Plot qualitatively the gain and phase diagrams for  $k_c G_p(s)$ . Indicate “corner” frequencies, asymptotic values of gain and phase angles and slope values.
- (10%) (b) Compute analytically the gain margin for  $k_c = 1$ .