National Exams December 2002

98-Elec-A1 Circuits

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

2. Candidates may use one of two calculators, a Casio FX-991 or Sharp EL-540. This is a closed-book examination. A short table of Laplace transforms is included.

3.. Any *five* questions constitute a complete paper. Only the *first five* questions as they appear in your answer book will be marked.

4. All questions are of equal value.

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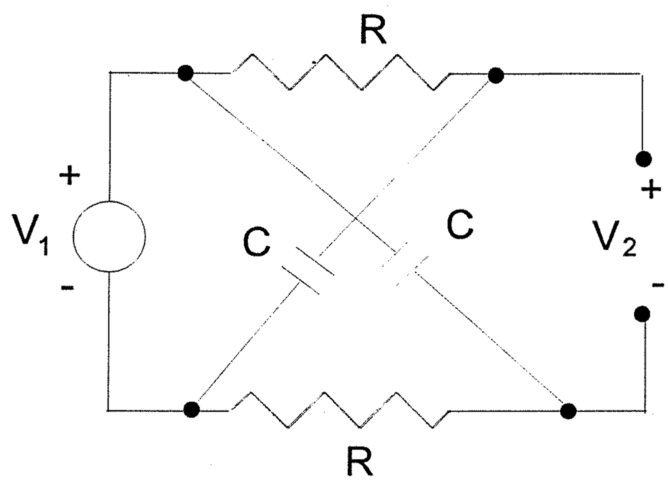
1. (a) Write the node voltage equations for the RC-lattice network shown below, and use them to derive the transfer function $\frac{V_2(s)}{V_1(s)}$ if $R = 10$ MΩ and $C = 0.1$ μF.



(b) Determine the response of this network to a unit impulse.

2 The transfer function of a network is given by

$$\frac{V_2(s)}{V_1(s)} = \frac{24(s^2+6s+25)}{(s+3)(s^2+4s+5)}$$

(a) Determine $v_2(t)$ if $v_1(t)$ is a unit step, assuming zero initial conditions.

(b) Determine the steady-state component of $v_2(t)$ if

$$v_1(t) = 5 + 3\cos\left(2t + \frac{\pi}{3}\right).$$

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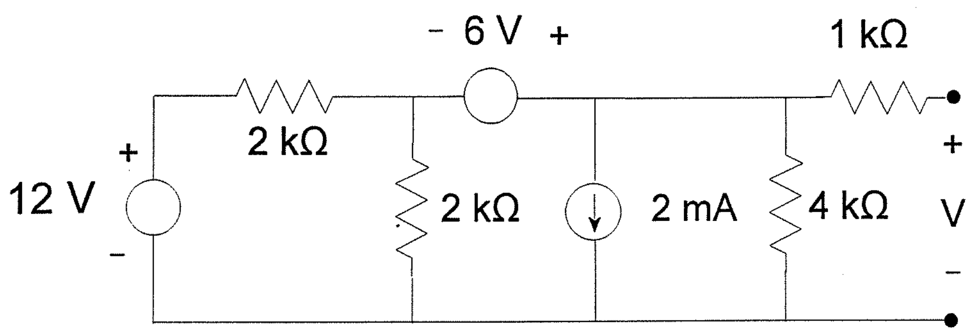
3. A network with two voltage sources, and a current source is shown below.



(a) Determine the Thevenin equivalent circuit at the two terminals emerging from the network.

(b) What should be the value of the load connected at these terminals so that the maximum possible power is delivered to this load?

(c) What will be the value of this maximum power?

4. The Laplace transform of the function $f(t)$ is given by

$$F(s) = \frac{15(s+2)(s+4)}{(s+3)(s^2+4s+20)}$$

Determine the Laplace transform of each of the following

(a) $e^{-3t}f(t)$, (b) $tf(t)$, (c) $\frac{df}{dt}$.

5. Determine the inverse Laplace transform of each of the following:

(a) $\frac{12(s+2)}{s(s+1)^2}$, (b) $\frac{10(s+1)}{(s+2)(s^2+2s+2)}$

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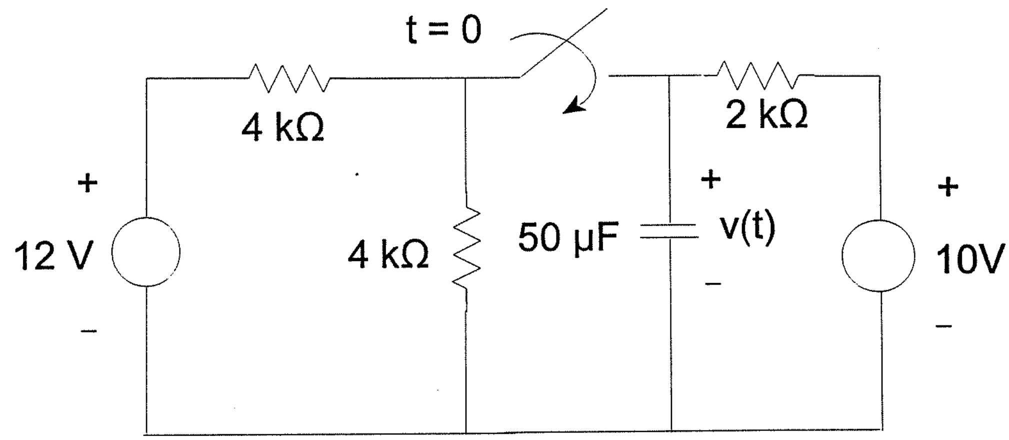
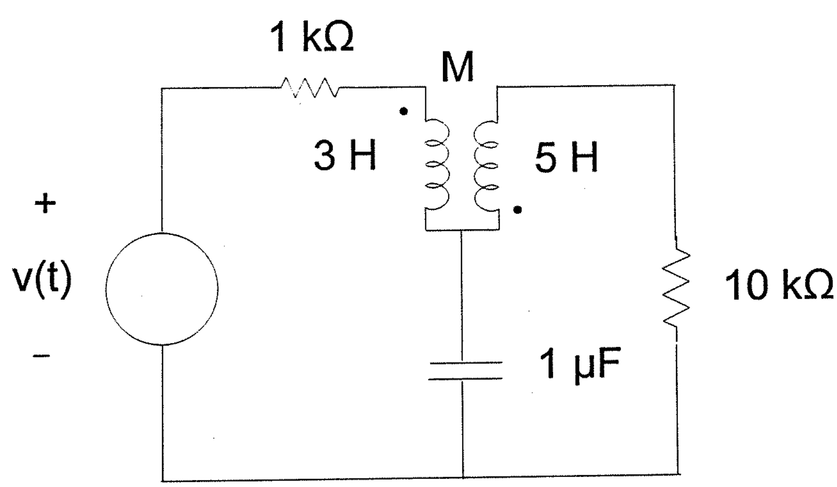
6. In the network shown below, the switch is closed at $t = 0$. Determine, $v(t)$, the voltage across the capacitor for $t \geq 0$ if it is known that $v(0) =$ 5 volts.



7. In the network shown below, the mutual inductance M between the coils is 2 H and the applied voltage is given by $v = 100 \cos 1000t$. Determine the steady-state voltage across the 10 kΩ resistor.



END

A SHORT TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s)$	$f(t)$	$F(s)$
unit impulse	1	$\sin\beta t$	$\frac{\beta}{s^2+\beta^2}$
unit step	$\frac{1}{s}$	$e^{-\alpha t}\cos\beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}\sin\beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
t	$\frac{1}{s^2}$		
$\cos\beta t$	$\frac{s}{s^2+\beta^2}$	$tf(t)$	$-\frac{dF(s)}{ds}$

INVERSE LAPLACE TRANSFORMATION

Given any proper rational function $F(s)$, perform partial fraction expansion by evaluating residues at the various poles. Inverse Laplace transform for each term can now be obtained using the following table,

$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$e^{-\alpha t}(2C\cos\beta t+2D\sin\beta t)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\frac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!}(C\cos\beta t+D\sin\beta t)$