National Exams - Dec 2002
98-Elec-A2, Control
3 hours duration

NOTES:

1. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models. This is a Closed Book examination. However, Candidates are permitted to bring a double-sided, 8.5 by 11 formula sheet.
3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. Suppose that the following open loop transfer function

$$G(s) \;=\; K\frac{(s+2)(s^2+4s+5)}{s(s+5)(s^2-4s+5)}$$

represents the balance control of a human on roller blades. K is a positive gain that is proportional of the ability of the human to react to changes in the system input. The human balance control operates in a unity feedback configuration.

   (a) Sketch a root locus of the closed loop system poles for positive K on the graph paper provided. (Accurate $j\omega$ axis crossings are not required, but computation of angles of departure/arrival are.)

   (b) *From your sketch*, estimate the value of K for which balance is lost, and the human begins to wobble, and report the frequency of the wobble.

   (c) Determine the resulting closed loop system steady state response to step and ramp input signals for K large enough to ensure stability, (i.e, this simulates the ability of the human to react to step changes in velocity and acceleration, induced by going up and down hills.).

98-Elec-A2 Control

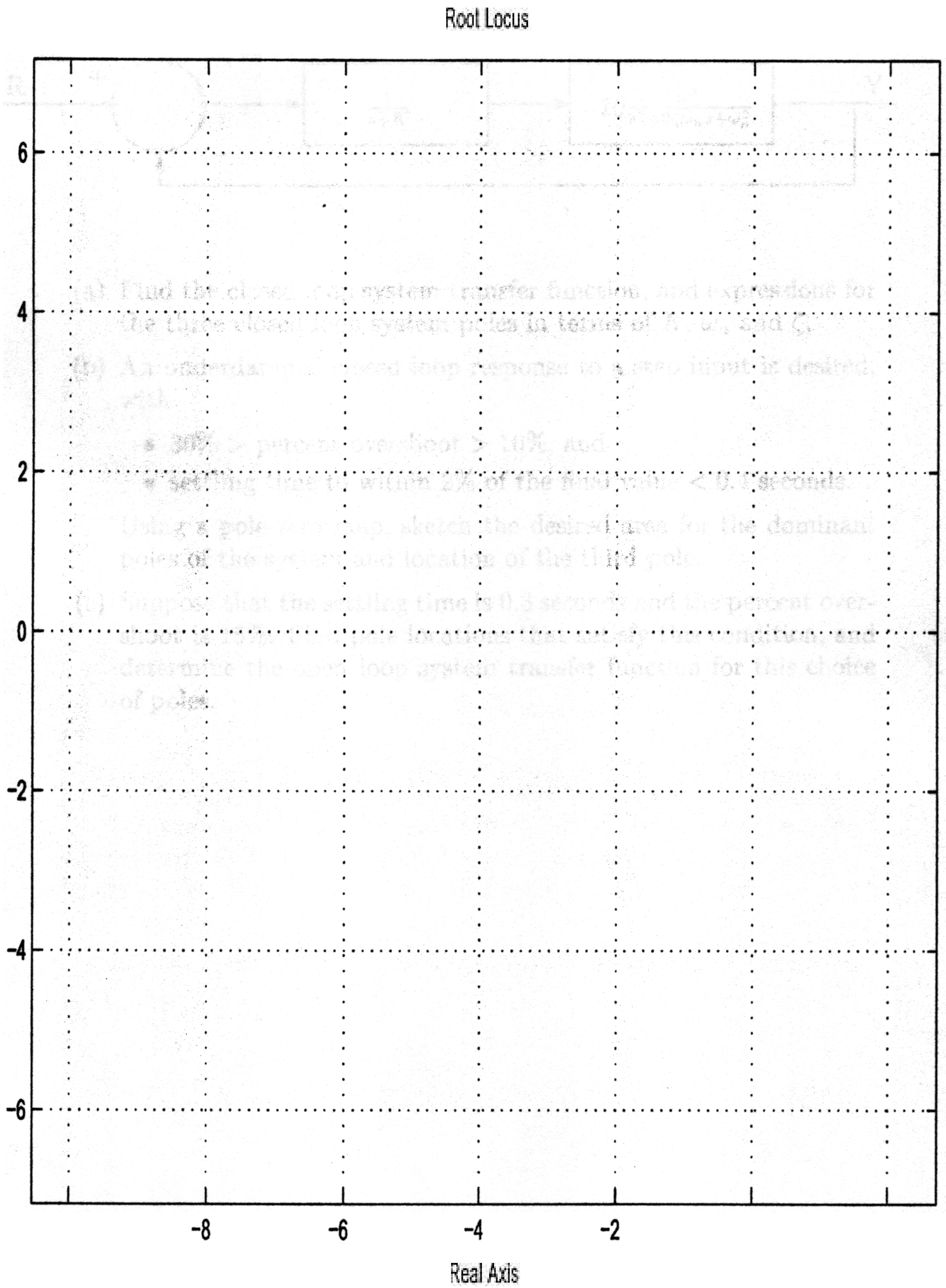
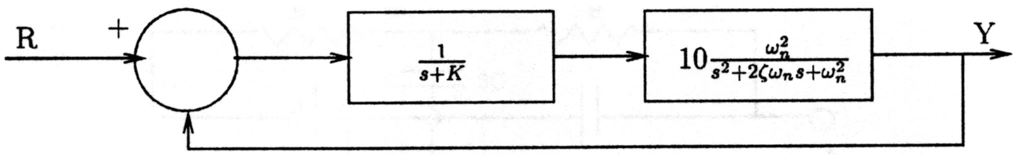


Figure Root Locus Graph Paper for Question

2. Consider the following system under unity feedback control:



(a) Find the closed loop system transfer function, and expressions for the three closed loop system poles in terms of K, ω_n and ζ.

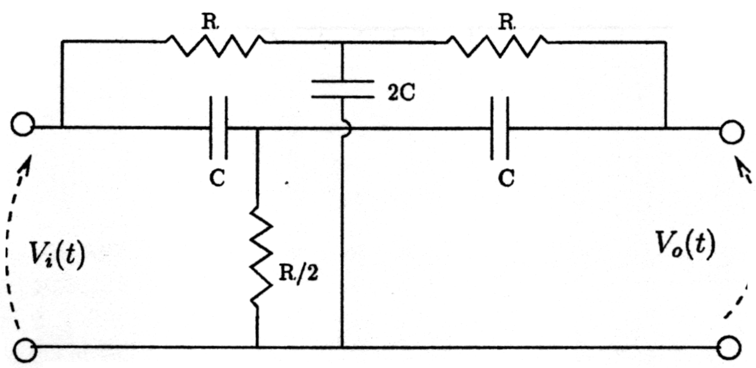
(b) An underdamped closed loop response to a step input is desired, with

- 30% > percent overshoot > 10%, and
- settling time to within 2% of the final value < 0.4 seconds.

Using a pole-zero map, sketch the desired area for the dominant poles of the system and location of the third pole.

(c) Suppose that the settling time is 0.3 seconds and the percent overshoot is 15%. Find pole locations that satisfy this condition, and determine the open loop system transfer function for this choice of poles.

3. Consider the following block diagram of a twin-T network:



(a) Show that the transfer fucntion of the network is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{(s\tau)^2 + 1}{(s\tau)^2 + 4s\tau + 1}; \tau = RC$$

(b) Sketch the corresponding Bode plot for the network on the graph paper provided (as a function of $1/\tau$).

(c) Suppose that the following signal is applied to the circuit:

$$V_i(t) = 10 + \sin(t) + \cos(8t)$$

Find the value of the constant τ such that the frequency component with $\omega = 1$ is maximally attenuated, and calculate the steady state response $V_{o_{ss}}(t)$ to the input $V_i(t)$.

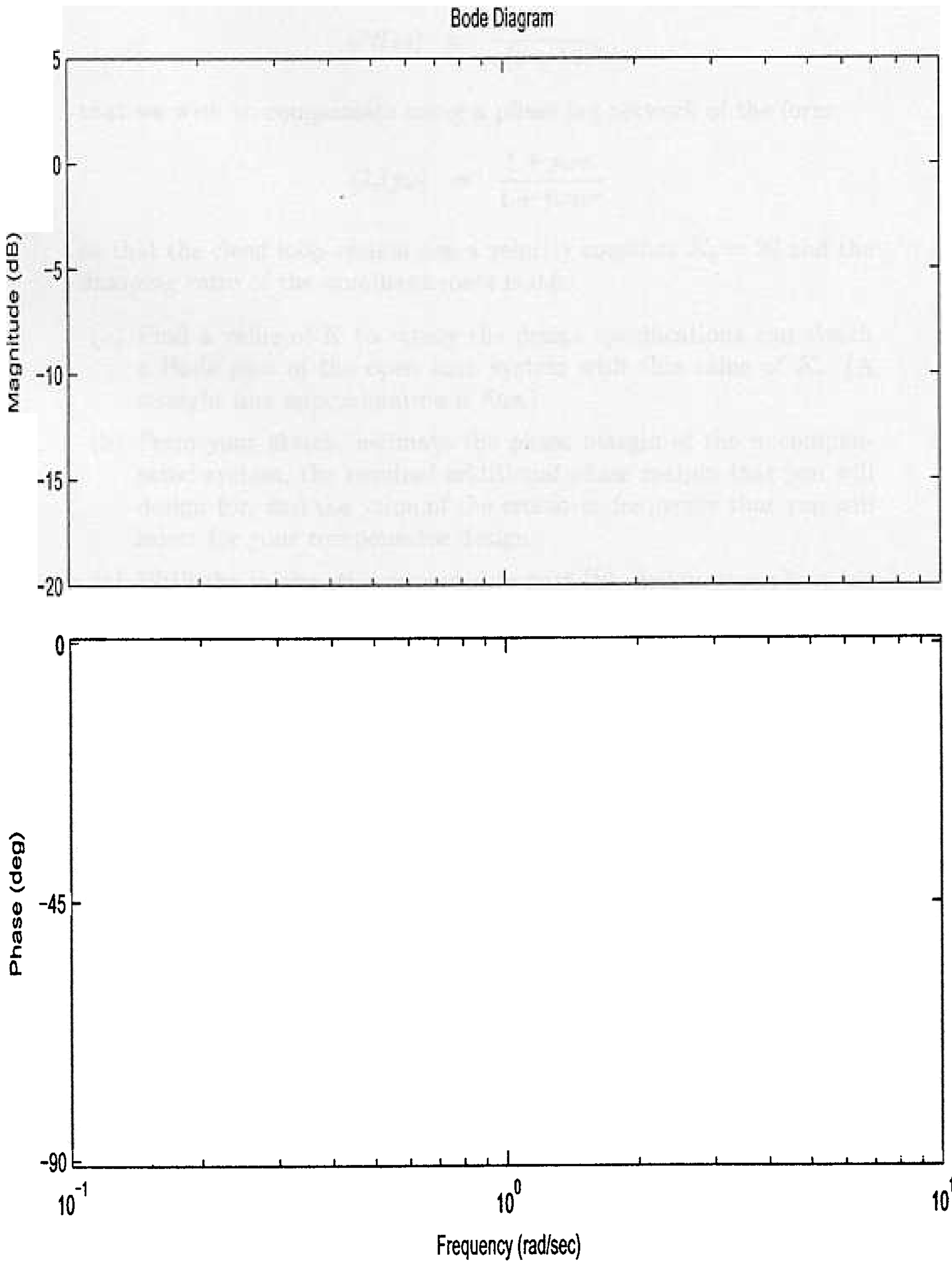


Figure 2: Bode Plot Graph Paper for Question 3

4. Suppose that we have a system with loop transfer function given by

$$GH(s) \qquad \frac{K}{s(s+10)^2}$$

that we wish to compensate using a phase lag network of the form

$$G_c(j\omega) \qquad \frac{1+j\omega\tau}{1+j\omega\alpha\tau}$$

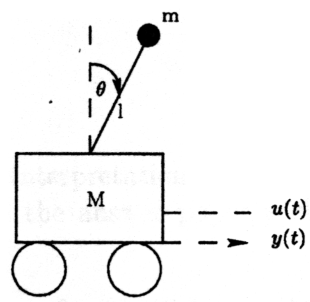
so that the closd loop system has a velocity constant K_v 20 and the damping ratio of the dominant roots is 0.6.

(a) Find a value of K to satisfy the design specifications and sketch a Bode plot of the open loop system with this value of K. (A straight line approximation is fine.)

(b) From your sketch, estimate the phase margin of the uncompensated system, the required additional phase margin that you will design for, and the value of the crossover frequency that you will select for your compensator design.

(c) With the information provided in part (b), design your phase lag compensator, and report the resulting values for τ and α.

5. Suppose that we have a cart and inverted pendulum system, as shown in the figure: Under the assumptions $M >> m$, θ small, and letting



$(x_1, x_2, x_3, x_4) = (y, \dot{y}, \theta, \dot{\theta})$, the following force balance equations may be derived:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{mg}{M}x_3 + \frac{1}{M}u(t) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{g}{l}x_3 - \frac{1}{Ml}u(t)
\end{aligned}$$

(a) Find the characteristic equation for this system and determine if it is stable.

(b) Suppose that we are able to measure θ and $\dot{\theta}$. What are the requirements on the positive proportional and derivative controller gains K_1 and K_2, i.e.,

$$u(t) = K_1\theta + K_2\dot{\theta}$$

so that the system is marginally stable?

(c) Suppose that we are interested in tracking a reference signal $\theta_{ref}(t)$. Using the following control signal,

$$u(t) = K_1(\theta(t) - \theta_{ref}(t)) + K_2(\dot{\theta}(t)),$$

find the closed loop system transfer function $\frac{\theta(s)}{\theta_{ref}(s)}$.