NATIONAL EXAMS **December 2002**

98-Elec-B2, Digital Signal Processing

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use one of two calculators, a Casio FX-991 **or** Sharp EL-540. This is an **Open book** exam.
3. There are eight questions in this paper. Any **five** questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. A system has the following transfer function:

$$H(z) = \frac{1}{(1+0.6z^{-1})(1-1.1z^{-1})}.$$

   (a) Specify an appropriate Region Of Convergence (ROC) to make the system stable.

   (b) Based on your answer in part (a), determine the impulse response of the system.

   (c) The system cannot be implemented exactly for real-time processing. Why? Suggest an approximation technique to implement such a system.

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2. Consider an all-pass reverberator with the following transfer function:

$$H(z) = \frac{-0.8 + z^{-50}}{1 - 0.8z^{-50}}.$$

   (a) Show that the reverberator has a flat magnitude response of unity. That is, $|H(e^{j\omega})| = 1$ for all ω.

   (b) If $\{x(n)\}$ is the input sequence to the reverberator, express the output sequence in terms of the input sequence in the time interval $110 \leq n \leq 130$. (Hint: determine the first few non-zero valued impulse response samples and use convolution!)

   (c) With $x(n) = (\cos(0.25\pi n) + 2\cos(0.3\pi n))u(n)$ as the input, determine the output sequence at the steady state. $u(n)$ is a unit step sequence,

$$u(n) = \begin{cases} 1, & n \geq 0; \\ 0, & n<0. \end{cases}$$

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3. Consider the impulse response of a causal IIR filter $h(n) = \{1.0, 0.5, 0.2, 1.0, 0.5, 0.2, \ldots\}$, where the dots indicate the periodic repetition of the three samples $\{1.0,\ 0.5,\ 0.2\}$

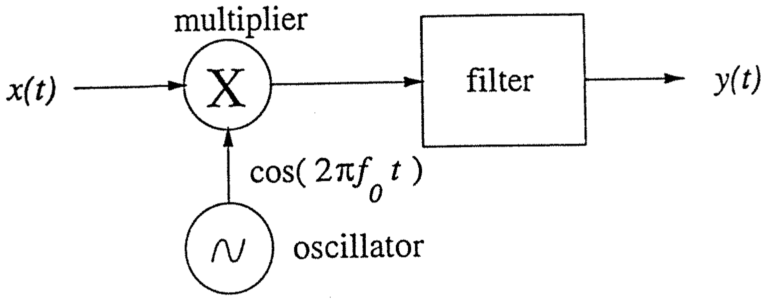
   (a) Determine the difference equation of the filter.

   (b) Determine the transfer function.

   (c) Determine the output of the system at $n = 49$ corresponding to the input $x(n) = \delta(n-2)$, where

$$\delta(n) = \begin{cases} 1, & n = 0; \\ 0, & \text{otherwise}. \end{cases}$$

4. Consider the following frequency-translation system.



The input $x(t)$ and the output $y(t)$ are sinusoidal waveforms with frequencies of 1 kHz and 9 kHz, respectively. The oscillator frequency f_0 is 8 kHz. The system is to be implemented in DSP with the underlying sampling frequency of 48 kHz.

(a) If the digital oscillator is a second-order IIR filter, determine its pole locations.

(b) Determine the frequency components of the signal at the output of the multiplier.

(c) Let the filter in the above diagram be implemented as a second-order IIR filter. If the poles of this IIR filter have a magnitude of 0.8, determine the appropriate pole and zero locations of the filter.

5. Consider the following two sequences:

$$\{x(n)\} = \{2,\ 1,\ 1,\ -2,\ 3,\ -1\}$$

$$\{h(n)\} = \{1,\ 0,\ -1\}.$$

(a) Determine the linear convolution of $\{x(n)\}$ and $\{h(n)\}$.

(b) Use the overlap-add method to determine the linear convolution. Partition $\{x(n)\}$ into three contiguous non-overlapping blocks. Each block contains two samples. Use FFT/IFFT (fast convolution) to process each block. All the computational details (e.g. signal flow graphs) of the required FFTs must be shown explicitly.

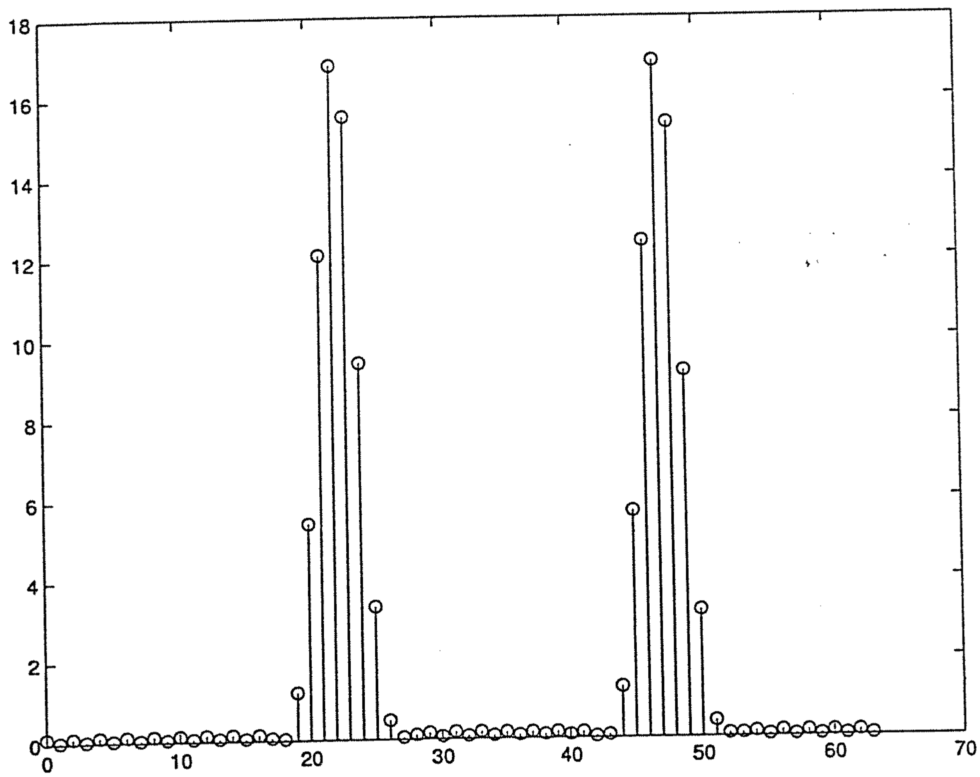
6. A dual-tone multi-frequency (DTMF) transmitter (touch-tone phone) encodes each key-press as a sum of two sinusoidal tones, with one frequency taken from group A and one group B, where:

$$\begin{aligned} \text{group A} &= 697,\ 770,\ 852,\ 941\ \text{Hz} \\ \text{group B} &= 1209,\ 1336,\ 1477\ \text{Hz.} \end{aligned}$$

A digital DTMF receiver collects the received DTMF samples at a sampling rate 4 kHz and computes the spectrum using FFT to determine the two frequencies that are present, and thus, the key that was pressed.

(a) What is the smallest number of time samples N that we should collect in order for the group-A frequencies to be resolvable from the group-B frequencies? Assume Hamming window is used prior to the FFT computation. (The 3-dB main lobe width of the Hamming window is $4\pi/N$)

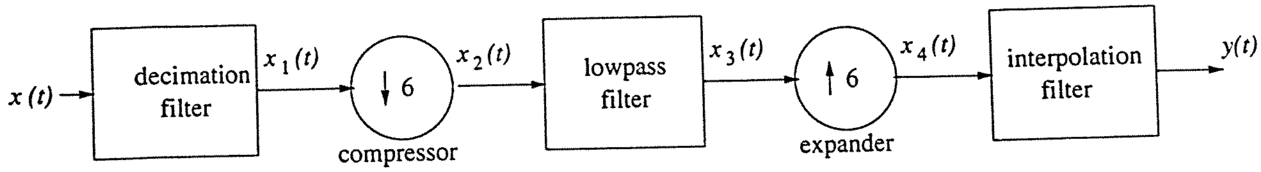
(b) Suppose that 64 samples of a DTMF signal are collected and the 128-point FFT is computed using zero padding technique. The following figure shows the resulting magnitude spectrum for $0 \leq k \leq 63$, where k is the FFT index.



From the spectrum, determine the two DTMF frequencies that are present in the samples.

(c) Suppose zero padding **is not** used in part (b), at which FFT indices would you expect to see the peaks. Sketch and label the corresponding magnitude spectrum.

7. The following figure illustrates the basic structure of a multirate lowpass FIR system.



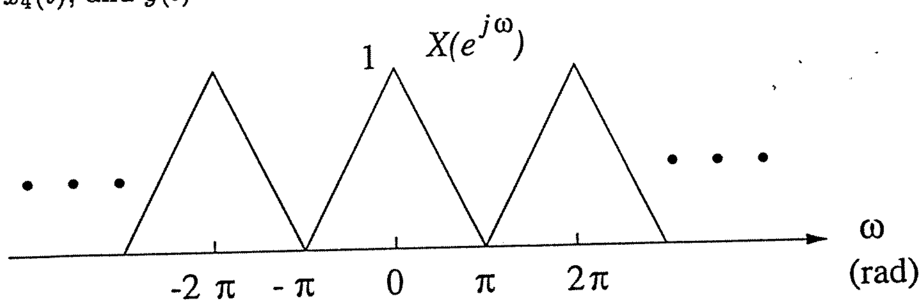
The overall frequency specifications of the system is given as follows:

- passband edge = 2.4 kHz.
- passband attenuation ≤ 2 dB.
- stopband edge = 2.8 kHz.
- stopband attenuation ≥ 50 dB.

The underlying sampling frequency of the input signal $x(n)$ is 48 kHz.

(a) Determine the appropriate frequency specifications of the decimation, lowpass, and interpolation filters.

(b) Suppose that $X(e^{j\omega})$ (the Fourier transform of $x(t)$) is as shown, and that the decimation, lowpass, and interpolation filters are all ideal filters with cutoff frequencies of $\pi/6$, $\pi/8$, and $\pi/6$, respectively. Sketch the Fourier transforms of $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, and $y(t)$.



Restrict your sketches to the digital frequency range $-2\pi \leq \omega \leq 2\pi$. Your sketches should indicate both the digital frequency ω (in radians) and the analog frequency f (in Hz).

8. A Digital filter has the following transfer function:

$$H(z) = \frac{1 - 1.8z^{-2} + z^{-4}}{1 - 0.4z^{-4}}.$$

The filter is implemented in a DSP chip.

(a) Draw the canonical realization of the filter. Write the corresponding I/O difference equations.

(b) If the filter is implemented based on the canonical form in part (a), what is the minimum amount of memory that is required to store the state variables?

(c) If the DSP chip uses a fractional data representation for all Data ALU (Arithmetic Logic Unit) operations, how would you modify the I/O difference equation in part (a), and how would the state variables be processed differently?