NATIONAL EXAMS December 2002

98-Elec-B3 Advanced Control Systems

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

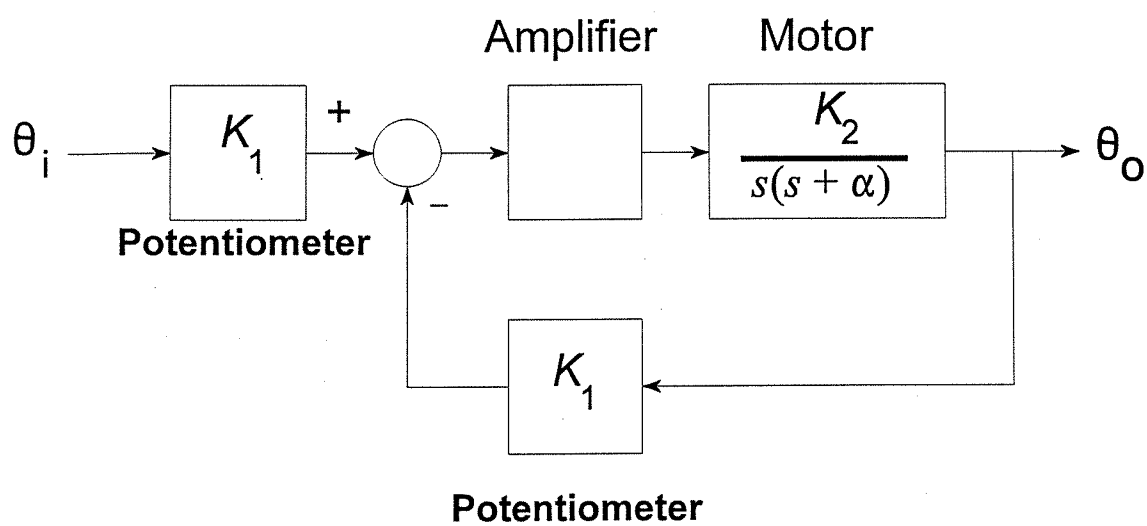
2. Candidates may use one of two calculators, a Casio FX-991 or Sharp EL-540. This is a closed-book examination. Tables of Laplace and z-transforms are included..

3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

... continued on page 2

1. A position control system is to be designed with 20% maximum overshoot in its step response and settling time of 2 seconds. You have on hand an amplifier with a variable gain K_a and a pole at -20, and a power amplifier of unit gain to drive the motor. Two ten-turn potentiometers are available for converting shaft position into voltage. A voltage of $\pm 5\pi$ volts is placed across the potentiometers. A dc motor of transfer function $\frac{K_2}{s(s+\alpha)}$ is also available. To experimentally determine the transfer function of the motor, a step input of 10 volts is applied to the armature of the motor with the geared load. It was found that the motor reached a constant speed of 100 radians per second, and it took 0.5 second to reach 63.2 % of this speed.



(a) Determine the transfer function of each component specifying the value of K_a so that the system operates with maximum overshoot of 20 % in response to a step input.

(b) What will be the steady-state error to a unit ramp input?

2. Consider the system described by the state equations

$$\dot{x} = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x$$

...continued on page 3

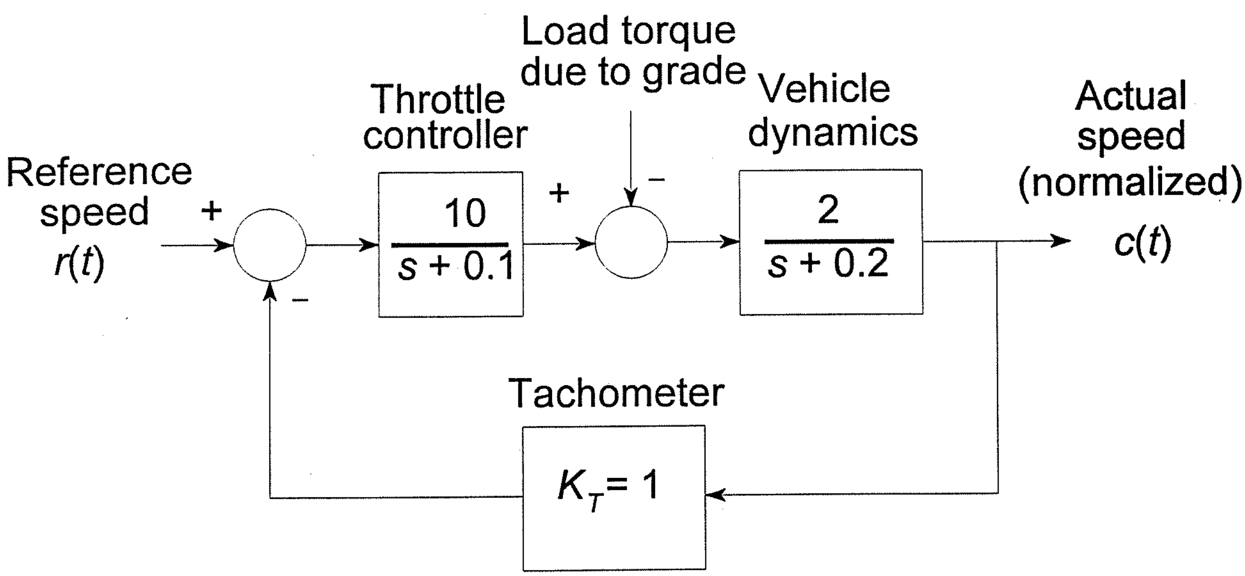
(a) It is desired to apply state-variable feedback according to the following equation:

$$u = K(r - k^T x)$$

Determine the values of the gain constant K and the state feedback vector k so that the poles of the closed-loop system will be located at -10 and $-3 \pm j4$, and the steady-state error to a step input will be zero.

(b) For the values of K and k determined in part (a), what will be the steady-state error of the system to a unit ramp input?

3. The block diagram of the cruise control system for an automobile is shown below, where $c(t)$ is the normalized speed.



(a) Determine the steady-state error in the speed when the reference input is a unit step corresponding to the normalized value of the desired speed. Express this error as percentage relative to the desired speed. Assume zero load torque due to grade.

(b) With the automobile moving at this steady speed along a level road, suddenly the grade changes. This corresponds to a load torque disturbance of 0.1 times the unit step. Find the new steady-state error in the speed as a percentage of the desired speed.

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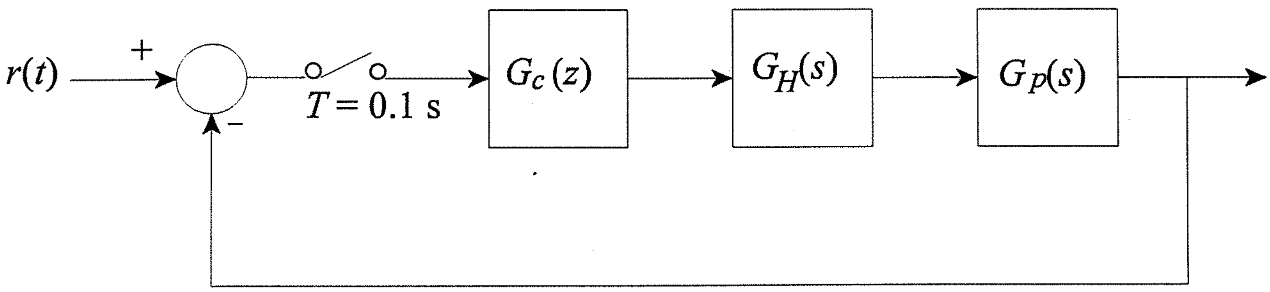
4. Consider the discrete-time control system shown in the following figure, where $G_h(z)$ is a zero-order hold and

$$G_p(s) = \frac{4}{s(s+2)}$$



Determine the transfer function of the digital controller $G_c(z)$ so that the system may have dead-beat response.

5. The following table shows a set of input-output data for a device represented by the equation

$$y = A e^{bx}$$

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	10.0	9.05	8.18	7.42	6.71	6.06	5.47	4.97	4.49	4.07	3.68

It is known that the measurements are contaminated with zero-mean white noise.

Obtain the best estimates for A and b in the least squares sense.

6. Consider a system described by the transfer function

$$G(s) = \frac{s+1}{s(s+4)(s+6)}$$

(a) Obtain a set of state equations for the system. Only the output is available for measurements.

(b) Design a minimal-order asymptotic state observer for the system. All the poles of the observer are to be located at -10.

END

A SHORT TABLE OF LAPLACE AND z TRANSFORMS

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{-\alpha T}}$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta T)}{z^2-2z\cos \beta T+1}$
$\sin\beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z \sin\beta T}{z^2-2z\cos \beta T+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha T}\cos \beta T)}{z^2-2ze^{-\alpha T}\cos \beta T+e^{-2\alpha T}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha T}\sin \beta T}{z^2-2ze^{-\alpha T}\cos \beta T+e^{-2\alpha T}}$
$tf(t)$	$-\frac{dF(s)}{ds}$	$-zT\cdot\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha T})$

INVERSE LAPLACE TRANSFORMATION

Given any proper rational function $F(s)$, perform partial fraction expansion by evaluating residues at the various poles. Inverse Laplace transform for each term can now be obtained using the following table,

$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$e^{-\alpha t}(2C\cos\beta t+2D\sin\beta t)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\dfrac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}(C\cos\beta t+D\sin\beta t)$

INVERSE z-TRANSFORMATION

Given $F(z)$, we perform partial fraction expansion of $F(z)/z$ and then multiply by z to get z back into the numerator. The following table can then be used for obtaining the inverse z-transform for each term, where complex poles have been expressed in the polar form. Note that we get only the values at the sampling instants, i.e., $f(nT)$.

$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{(C+jD)z}{z-re^{j\phi}}+\dfrac{(C-jD)z}{z-re^{-j\phi}}$	$2r^{n}(C\cos n\phi-D\sin n\phi)$
$\dfrac{Kz}{(z-a)^{r}}\quad r=2,3,\cdots$	$\dfrac{Kn(n-1)\cdots(n-r+2)}{(r-1)!\,a^{r-1}}a^{n}$