National Exams May 2002

98-Elec-A1 Circuits

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

2. Candidates may use one of two calculators, a Casio FX-991 or Sharp EL-540. This is a closed-book examination. Tables of Laplace transforms will be provided.

3.. Any *five* questions constitute a complete paper. Only the *first five* questions as they appear in your answer book will be marked.

4. All questions are of equal value.

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1. The transfer function of a network is given by

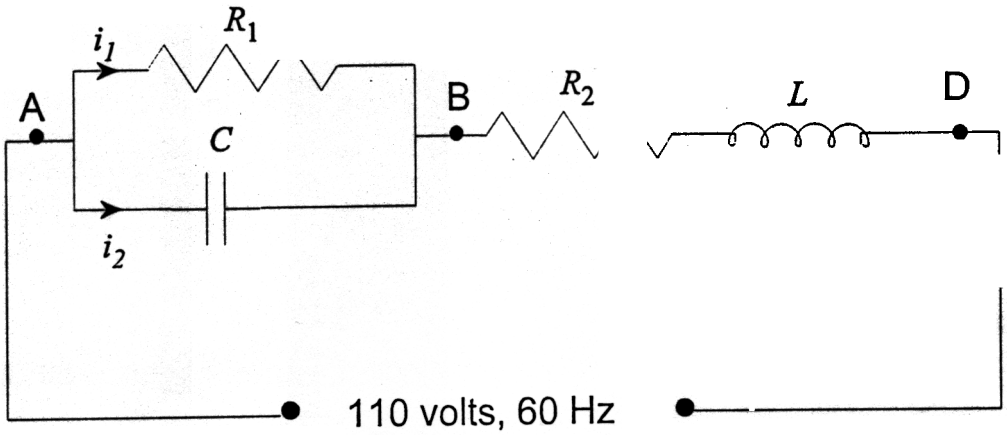
$$\frac{V_2(s)}{V_1(s)} = \frac{250(s^2+2s+4)}{(s+5)(s+10)^2}$$

Determine $v_2(t)$ if $v_1(t)$ is a unit step, assuming zero initial conditions.

Determine the steady-state component of $v_2(t)$ if

$$v_1(t) = 5 + 6\cos\left(3t + \frac{\pi}{4}\right)$$

2. Consider the network shown in the following figure.



The effective value of both the voltages V_{AB} and V_{BD} is 60 volts and the currents i_1 and i_2 have effective values of 12 and 5 amperes, respectively.

(a) Determine the values of the components, R_1, R_2, C and L in the network.

(b) What is the average power supplied by the power source, and what is the overall power factor?

3. The Laplace transforms of two time functions are given below. For each of them, determine the initial value, $x(0)$. and the final value, $\lim_{t\to\infty} x(t)$

(a) $X(s) = \dfrac{3s^2+2s+5}{s^3+4s^2+6s}$ (b) $X(s) = \dfrac{4s+5}{s^2+4}$

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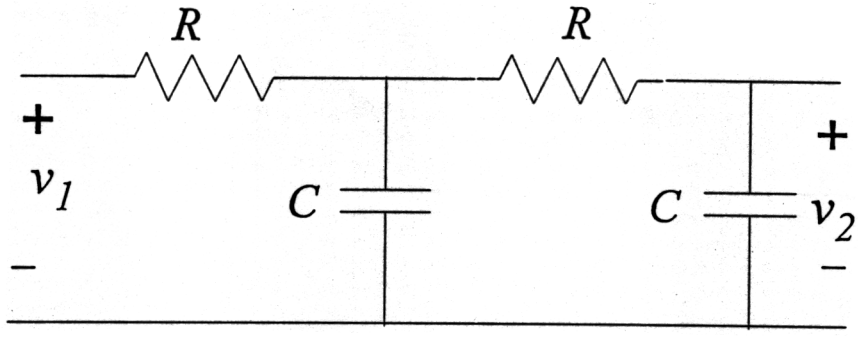
4. (a) Determine the transfer function relating $V_2(s)$ to $V_1(s)$ in the R-C network shown below, where R = 5 MΩ and C = 0.2 μF.



(b) If the input v_1 is a sinusoid, what must be its frequency so that the steady-state output v_2 differs in phase with v_1 by 90°?

5. Determine the Laplace transform of each of the following functions of time, where $u(t)$ is the unit step function.

(i) $x(t) = t^2 e^{-3t} u(t)$

(ii) $x(t) = \int_0^t e^{-3\tau} \cos 4\tau \, d\tau \; u(t)$

(iii) $x(t) = \frac{d}{dt}\left[3e^{-2t}\cos 4t\right] u(t)$

6. Two terminals emerge from a network containing an ac power source and passive circuit elements. The open circuit voltage measured at these terminals had an effective value of 100 volts. When a 6-ohm resistor was connected at the terminals, it drew a current of effective value 5 amperes. When a pure capacitor of impedance 8 ohms was connected at the terminals it drew a current of 10 amperes.

(i) Determine the Thevenin equivalent circuit for the network at these terminals.

(ii) What load impedance should be connected at these terminals so that the power output to the load is maximum?

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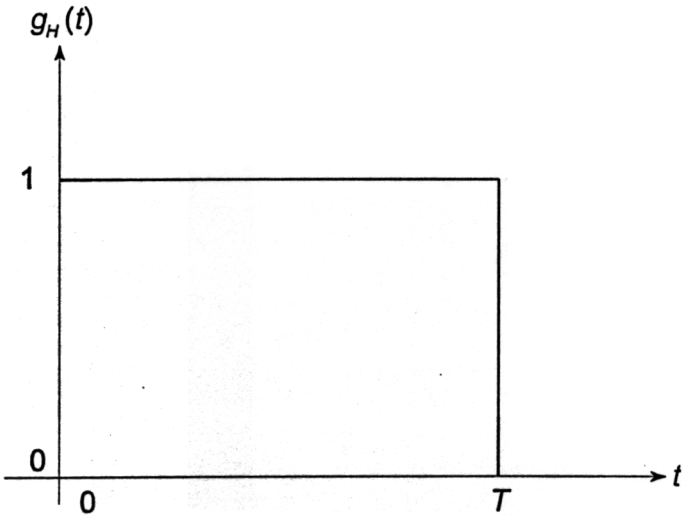
7. The impulse response of a zero-order hold, commonly used in sampled-data systems is shown below.



(i) Determine its transfer function

(ii) Determine and sketch its response to a unit step input.

END

A SHORT TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s)$
unit impulse	1
unit step	$\frac{1}{s}$
	$\frac{1}{s+\alpha}$
t	$\frac{1}{s^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$
	$\frac{\beta}{s^2+\beta^2}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
	$-\frac{dF(s)}{ds}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$

INVERSE LAPLACE TRANSFORMATION

Given any proper rational function $F(s)$, perform partial fraction expansion by evaluating residues at the various poles. Inverse Laplace transform for each term can now be obtained using the following table,

$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$e^{-\alpha t}(2C\cos\beta t+2D\sin\beta t)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\frac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!}(C\cos\beta t+D\sin\beta t)$