National Exams - May 2002
98-Elec-A2, Control
3 hours duration

NOTES:

1. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models. This is a Closed Book examination. However, Candidates are permitted to bring a double-sided, handwritten, 8.5 by 11 formula sheet.
3. Any four questions constitutes a complete paper. Only the first four questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. Consider the block diagram describing a system under proportional-integral control, as shown in the following Figure:

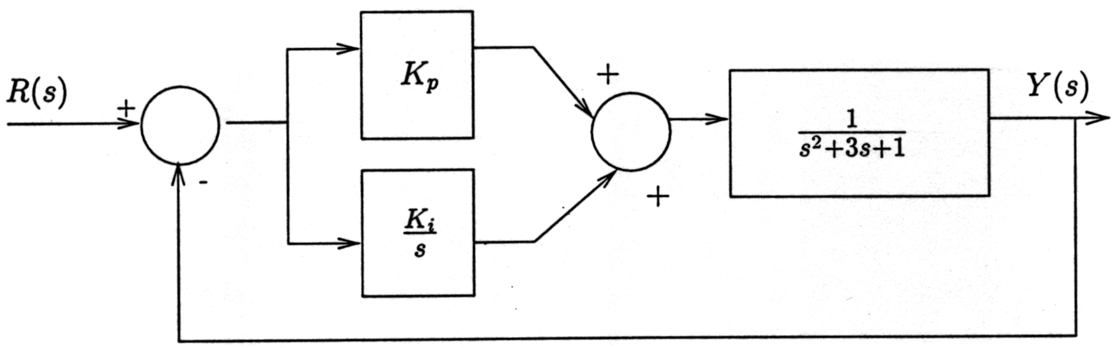

Figure 1: System Under PI Control

(a) Find the required constraints on K_p and/or K_i so that the steady state error to an input of $r(t) = 3 + 2t; t \geq 0$ is less than 0.1.

(b) Use the Routh-Hurwitz criterion to determine a range of K_p and K_i so that both closed loop system stability as well as the steady state error requirement described in 1. are met.

(c) Suppose that the controller gains are set to $K_p = K_i = 45$. Find the closed loop system transfer function.

(d) With controller gains set to $K_p = K_i = 45$, the system exhibits an underdamped response to a step input. Describe how you would include derivative control to the system, and comment on the resulting effect on rise time, overshoot, and steady state error.

2. Consider a unity feedback control system, with open loop transfer function $K_pGH(s)$. Frequency response plots of $K_pGH(s)$ are shown below:

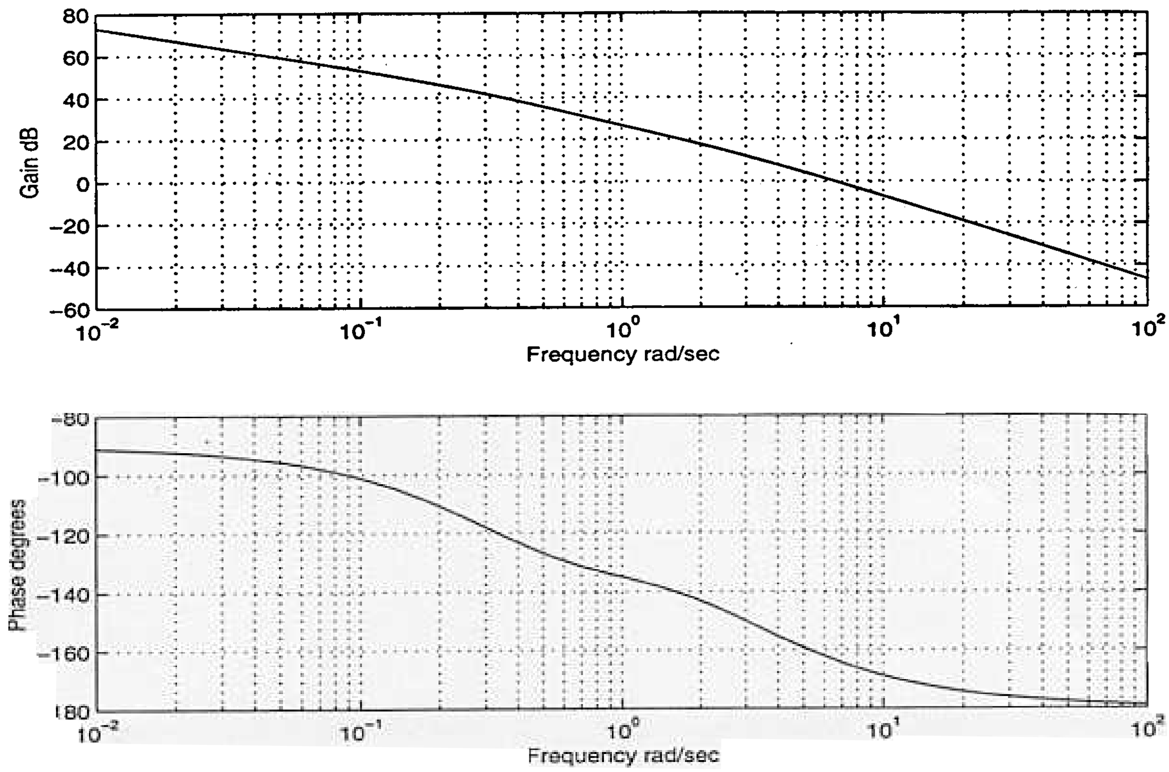

Figure 2: Frequency response plots of $K_pGH(s)$

(a) Measure the system gain margin, and use it to approximate how much of an increase in proportional control K_p the system can handle before becoming unstable.

(b) Based on the open loop frequency response plots, determine the system type number, and estimate the closed loop system steady state error to a unit step, unit ramp, and unit parabolic input.

(c) Based on the open loop frequency response plots, and assuming that the closed loop system exhibits predominantly second order characteristics, evaluate the closed loop system damping ratio, ζ, frequency of natural oscillations, ω_n, and DC gain, K_{dc}.

(d) Obtain an expression for a second order dominant pole transfer function to approximate the closed loop system.

3. A process has a transfer function

$$G(s) = \frac{5}{(s+1)(s^2+2s+2)}$$

and is included in a unity feedback loop with a proportional controller K_p in the feedforward branch of the control loop.

(a) Sketch a Nyquist contour for this system.

(b) Apply the Nyquist stability criterion to find a suitable range of K for stable closed loop system behaviour.

(c) Verify the above stability result using a Routh-Hurwitz analysis.

(d) From the Nyquist contour, estimate the phase margin, and briefly describe how it relates to stability of the system.

4 (a) Consider the open loop ($K_t = 0$) speed control of an armature controlled DC motor, subjected to a load disturbance $T_d(s)$, shown in the following Figure: Find the steady state deviation in speed due to a load disturbance $T_d(s) = \frac{D}{s}$.

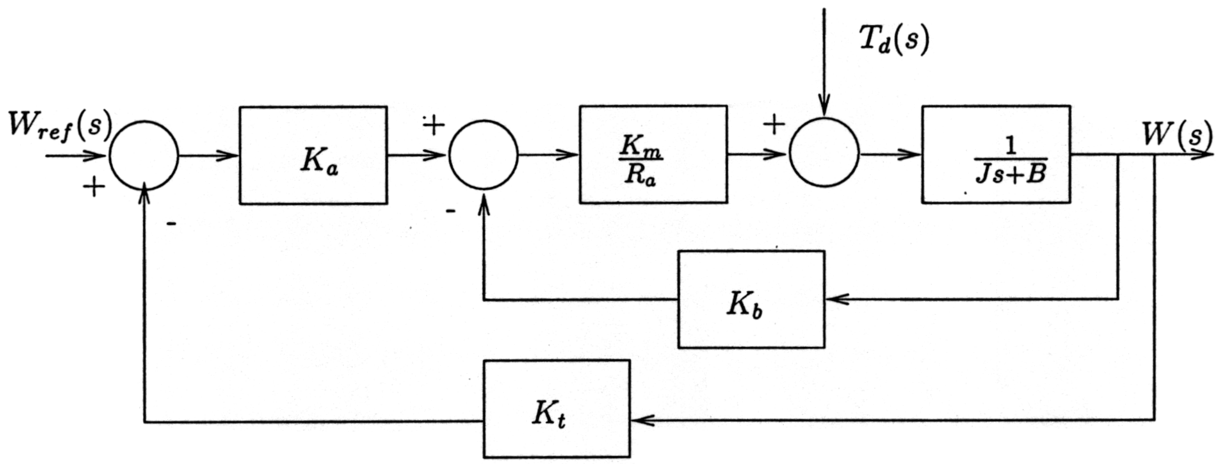

Figure 3: Speed Control of an Armature Controlled DC Motor

(b) Now consider closed loop ($K_t \neq 0$) operation, and obtain an expression for $W(s)$ due to a load disturbance $T_d(s)$.

(c) Determine the smallest amplifier gain that can be used so that the steady state error to a step load disturbance is less than 2%.

(d) Comment on the effect of the amplifier gain on disturbance rejection in both the open loop and closed loop configurations shown.

5. A unity feedback system has an actuator/process modeled by state space equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 0.03x_1 + 0.95u \\ y &= x_1 \end{aligned}$$

with $y(t)$ the system output and $u(t)$ the control signal

(a) Find the actuator/process transfer function, the poles and zeros, and determine if the system is stable.

(b) Suppose that the system controller has the form

$$u(t) = K_p y(t) + K_d \frac{d}{dt} y(t) - K_p y_R(t)$$

where $y_R(t)$ is the closed loop system reference input. Form the closed loop system state space model.

(c) Find the closed loop system transfer function from $Y_R(s)$ to $Y(s)$, and choose K_p and K_d so that the closed loop system has poles located at $-3 \pm j2$.

(d) Sketch the closed loop system response to a step change in $y_R(t)$, and compute and label the percent overshoot, the natural frequency of oscillation, and the steady state error.

6. Consider the following block diagram describing a system under proportional-integral-derivative control:

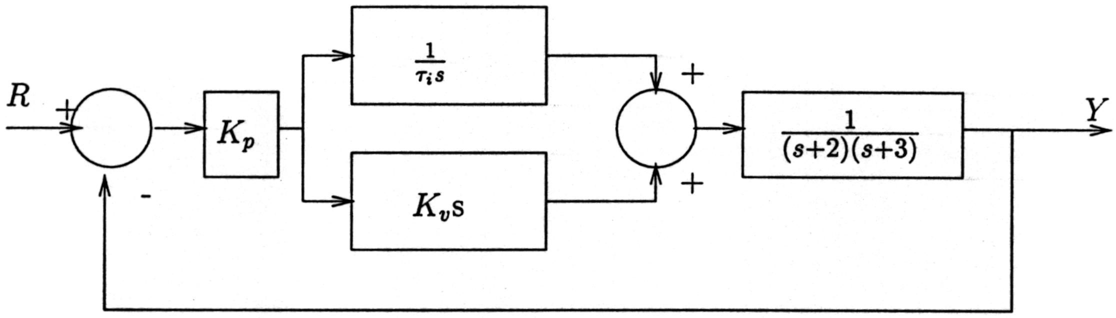

Figure 4: System Under PID Control

(a) Suppose that $\tau_i = 10, K_v = 1,$. Sketch a Root Locus for $0 < K_p < \infty$.

(b) From your sketch, find a pair of dominant poles so that the closed loop system response to a step input exhibits 10% overshoot.

(c) From your sketch, estimate the value of K_p at this location.

(d) With this value of K_p, determine the closed loop system transfer function.