NATIONAL EXAMS May 2002

98-Elec-B3 Advanced Control Systems

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

2. Candidates may use one of two calculators, a Casio FX-991 or Sharp EL-540. This is a closed-book examination. Tables of Laplace and *z*-transforms are included..

3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

... continued on page 2

1 The transfer function of a linear system is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{12(s+\alpha)}{(s+1)(s+2)(s+3)}$$

(a) Derive a set of state equations for this system in the observer canonical form.
(b) Sketch an analogue computer realization diagram for the above.
(c) Show that the state equations in this form are completely observable for all possible values of the parameter α.
(d) For what values of α will these equations cease to be completely controllable?

2. The transfer function of a linear system is given below.

$$\frac{Y(s)}{U(s)} = \frac{4s^2+6s+25}{s^3+10s^2+12s+180}$$

(a) Derive a set of state equations in the controller canonical form.
(b) Determine if this system is stable.
(c) It is desired to apply state-variable feedback according to the following equation:

$$u = K(r - \boldsymbol{k}^T \boldsymbol{x})$$

Determine the values of the gain constant K and the state feedback vector $\boldsymbol{k}$ so that the closed-loop system will have poles at -10 and $-3 \pm j4$, and the steady-state error to a step input will be zero.

(d) For the values of K and $\boldsymbol{k}$ determined in part (c), what will be the steady-state error of the system to a unit ramp input?

3. The control system for a cruise-control system can be represented by a unity-feedback system with the forward path transfer function

$$G_p(s) = \frac{K}{s(s+8)(s+10)}$$

The closed-loop system is required to satisfy the following specifications:

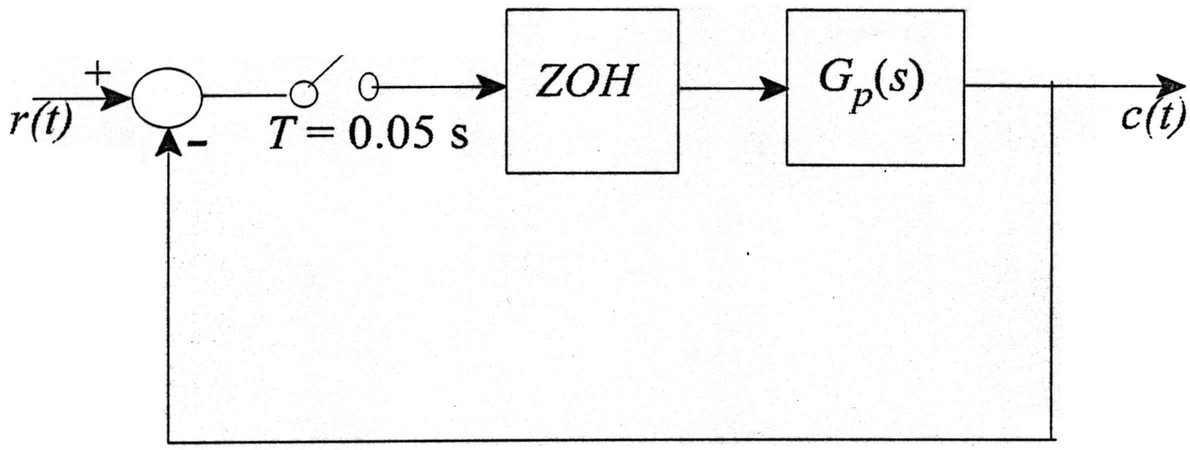
(a) The steady-state error to a unit ramp input should be 0.05.
(b) The damping ratio of the complex poles should be 0.5.
(c) The settling time of the system response should be less than 0.5 second.

Design a suitable compensator and verify that all the specifications are met.

continued on page 3

4. Consider a unity-feedback sampled-data system shown below , where *ZOH* represents a zero-order hold and

$$G_p(s) = \frac{K}{s(s+2)}$$



What is the maximum value of K for which the system will be stable?

What must be the value of K for which the system will be stable with gain margin equal to 6 dB?

For K determined in part (b) above, what will be the response of the closed-loop system to a unit step input?

5. The transfer function of a linear system is given by

$$\frac{Y(s)}{U(s)} = \frac{20}{s(s+2)(s+5)}$$

The input, $u(t)$, is sampled every 0.1 second, and held constant between the sampling instants.

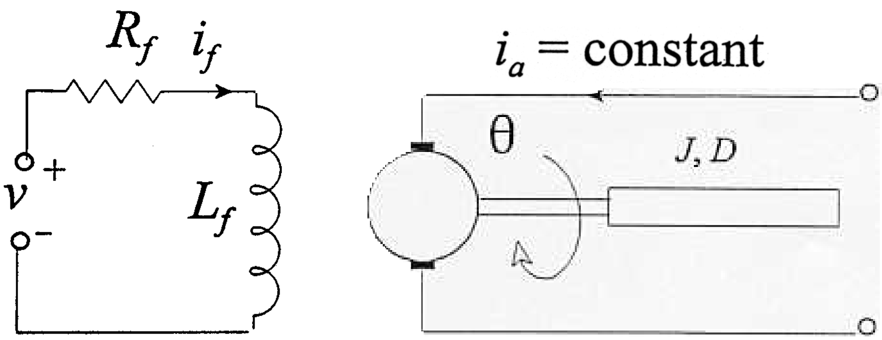
Selecting the output and its first two derivatives as the state variables, obtain the state transition equation of the resulting discrete-time system.

Verify that the system equations thus obtained are completely controllable.

... continued on page 4

(c) Calculate the input sequence which will transfer the state of the system from the origin of the state space to the point $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ in the minimum number of sampling intervals.

6. A field -controlled ds servomotor is shown below, where J represents the moment of inertia of the armature and the load, D is the damping coefficient, θ is the angular motion in radians and i_f is the field current.



(a) Derive the state equations using θ, $\dot{\theta}$ and i_f as the state variables.

(b) Derive the state equations using θ, $\dot{\theta}$ and $\ddot{\theta}$ as the state variables.

(c) Use the state equations derived in part (i) to determine the transfer function relating the Laplace transform of output $\theta(t)$ to that of the input $v(t)$.

END

A SHORT TABLE OF LAPLACE AND z TRANSFORMS

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
	$\frac{1}{\ }$	$\frac{z}{\ }$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta T)}{z^2-2z\cos\beta T+1}$
	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta T}{z^2-2z\cos\beta T+1}$
$e^{-\alpha t}\cos\beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha T}\cos\beta T)}{z^2-2ze^{-\alpha T}\cos\beta T+e^{-2\alpha T}}$
$e^{-\alpha t}\sin\beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha T}\sin\beta T}{z^2-2ze^{-\alpha T}\cos\beta T+e^{-2\alpha T}}$
	$-\frac{dF(s)}{ds}$	$-zT\cdot\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha T})$

INVERSE LAPLACE TRANSFORMATION

Given any proper rational function $F(s)$, perform partial fraction expansion by evaluating residues at the various poles. Inverse Laplace transform for each term can now be obtained using the following table,

$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$e^{-\alpha t}(2C\cos\beta t+2D\sin\beta t)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{}$
$\dfrac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\dfrac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}(C\cos\beta t+D\sin\beta t)$

INVERSE z-TRANSFORMATION

Given $F(z)$, we perform partial fraction expansion of $F(z)/z$ and then multiply by z to get z back into the numerator. The following table can then be used for obtaining the inverse z-transform for each term, where complex poles have been expressed in the polar form. Note that we get only the values at the sampling instants, i.e., $f(nT)$.

$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{(C+jD)z}{z-re^{j\phi}}+\dfrac{(C-jD)z}{z-re^{-j\phi}}$	$2r^{n}(C\cos n\phi-D\sin n\phi)$
$\dfrac{Kz}{}$ $\quad r=2,3,\cdots$	$\dfrac{Kn(n-1)\cdots(n-r+2)}{(r-1)!\,a^{r-1}}a^{n}$