National Exams May 2003

98-Elec-A1 Circuits

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

2. Candidates may use one of two calculators, a Casio FX-991 or Sharp EL-540. This is a closed-book examination. A short table of Laplace transforms is included.

3.. Any *five* questions constitute a complete paper. Only the *first five* questions as they appear in your answer book will be marked.

4. All questions are of equal value.

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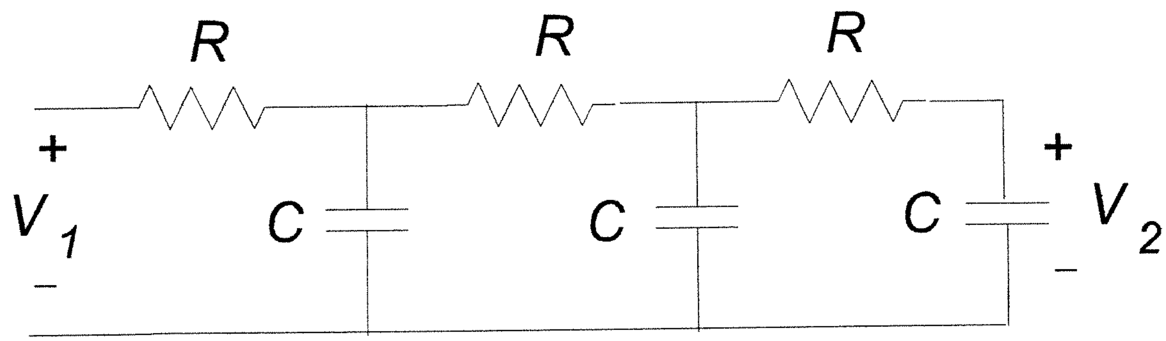
1. The RC-ladder network shown below can be used in the feedback path of an amplifier to implement an oscillator. This network produces a phase-shift of 180° at a certain frequency to give positive feedback.



(a) Determine the transfer function $\frac{V_2(s)}{V_1(s)}$ if $R = 2$ MΩ and $C = 0.1$ μF.

(b) At what frequency will the output differ in phase with the input by 180°?

(c) What must be the minimum gain of the amplifier to produce sustained oscillations?

2. The transfer function of a network is given by

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{4(s+1)(s^2+4)}{(s+2)(s^2+2s+2)}$$

(a) What will be the impulse response of this network?

(b) Determine the steady-state component of $v_2(t)$ if

$$v_1(t) = 4 + 5\cos(t + \frac{\pi}{3}) + 3\cos(2t + \frac{\pi}{4})$$

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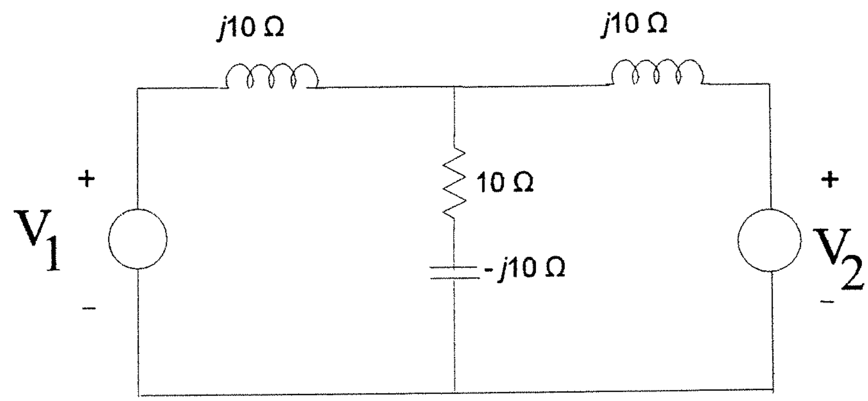
3. The following network shows two generators connected to a load through a lossless transmission line. Determine the power supplied by each generator if the effective value of the voltage supplied by each source is 100 volts and the voltage $v_2(t)$ leads $v_1(t)$ by 30°.



4. Determine the Laplace transform of each of the following functions of time, where $u(t)$ is the unit step function.

(i) $t^2 e^{-2t} \cos 3t \; u(t)$, (ii) $\int_0^t e^{-2\tau} \sin 3\tau \, d\tau \; u(t)$, (iii) $\frac{d}{dt}\left[t e^{-3t} \sin 2t\right] u(t)$

5. Solve the following differential equations using Laplace transforms, with initial conditions as specified.

(i) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 4\cos 3t, \quad y(0) = 1, \quad \frac{dy}{dt}(0) = 2$

(ii) $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 9\frac{dy}{dt} = 4, \quad y(0) = 2, \quad \frac{dy}{dt}(0) = 1, \quad \frac{d^2y}{dt^2}(0) = 0$

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6. A lossy coil (inductance L and resistance r) is connected in parallel with a pure resistor R across a 60 Hz 110-volt power supply. The effective value of the current taken by the coil was found to be 6 amperes and the current taken by the resistor was found to be 8 amperes. The combination drew a total current of effective value 11 amperes.

   Determine the values of L, r and R.

7. The impulse response of a linear time-invariant network is given by

$$h(t) = [5e^{-2t} + 2e^{-3t}\cos 4t]u(t)$$

   where $u(t)$ is the unit step function.

   (a) Determine its transfer function.

   (b) What will be the output of this network to the input

$$2 + 4e^{-3t}$$

   for $t \geq 0$? Assume zero initial conditions.

END

A SHORT TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s)$	$f(t)$	$F(s)$
unit impulse	1	$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$
unit step	$\frac{1}{s}$		
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$
t	$\frac{1}{s^2}$	$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$tf(t)$	$-\frac{dF(s)}{ds}$

INVERSE LAPLACE TRANSFORMATION

Given any proper rational function $F(s)$, perform partial fraction expansion by evaluating residues at the various poles. Inverse Laplace transform for each term can now be obtained using the following table,

$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$e^{-\alpha t}(2C\cos \beta t+2D\sin \beta t)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\frac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!}(C\cos \beta t+D\sin \beta t)$