National Exams - May 2003
98-Elec-A2, Control
3 hours duration

NOTES:

1. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models. This is a Closed Book examination. However, Candidates are permitted to bring a double-sided, 8.5 by 11 formula sheet.

3. Any four questions constitutes a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

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1. Consider a feedback control system under proportional control with forward and feedback transfer functions given by

$$G(s) = \frac{1}{(s+1)(s+2)}, \text{ and}$$
$$H(s) = \frac{4}{(s+10)},$$

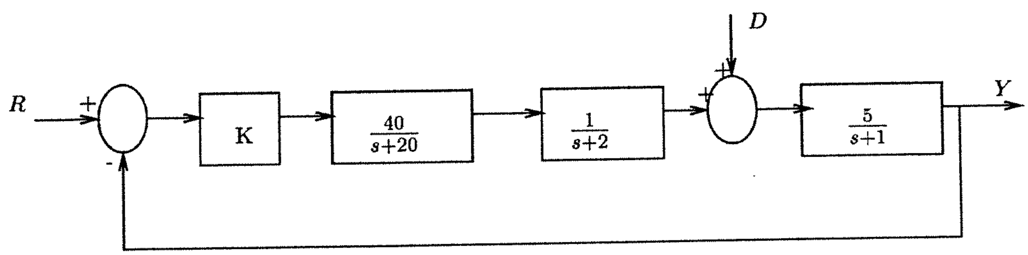
respectively.

(i) Use the Routh criterion to determine the range of positive proportional controller gain K for which the closed loop system is stable.

(ii) Suppose that we replace the proportional controller by a proportional-integral controller with transfer function $K(1+\frac{10}{s})$. Use the Routh criterion to determine the range of positive K for which the closed loop system is stable.

(iii) From your observations in (i) and (ii), can you (briefly) comment on the effect of adding the integral control with respect to system stability and step input tracking?

2. Consider the following system under feedback control:

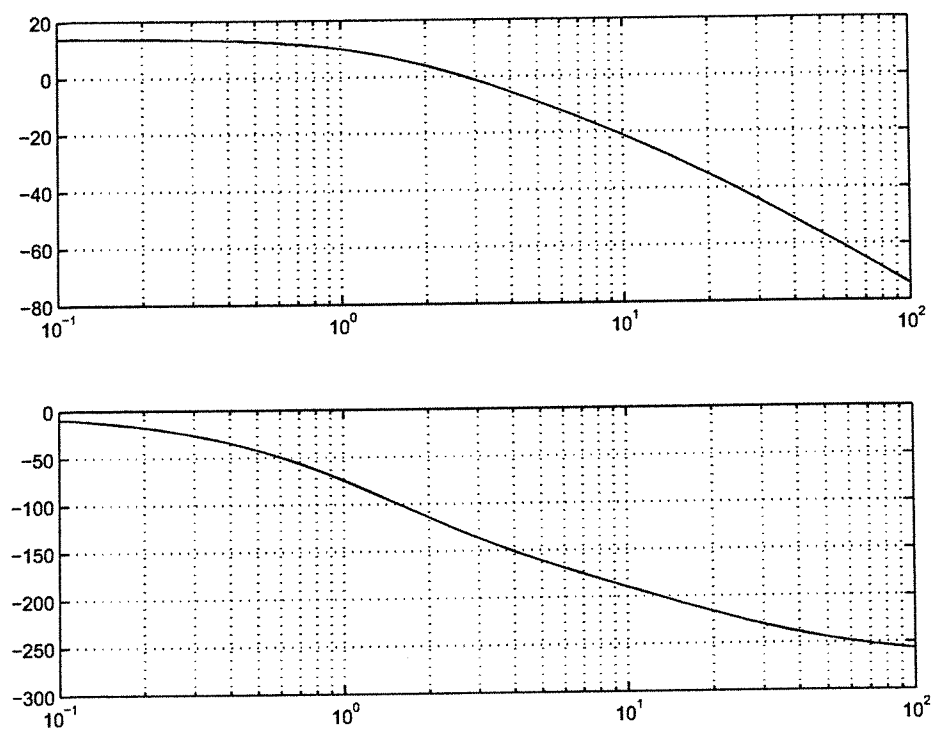


(i) Find the transfer function between input disturbance $D(s)$ and output $Y(s)$.

(ii) Suppose that the system is subjected to a typical input disturbance $D(s) = \frac{0.1}{s}$. Determine the magnitude of controller gain K needed to maintain the magnitude of steady state error of the system less than 0.1. (Assume that $R = 0$.)

(iii) Suppose that in addition to (ii), we want 1% steady state error in tracking step changes in the reference input $R(s)$. What is the range of K that will guarantee that both objectives will be met?

3. Consider again the system described in Question 2. A Bode plot is provided of the open loop transfer function of the system:

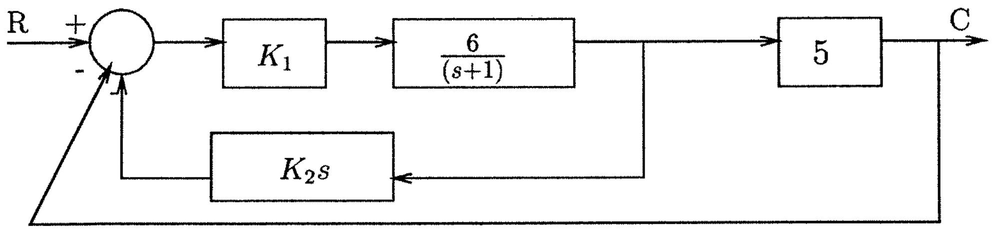


(i) Estimate the gain and phase margin of the system along with the corresponding critical frequencies.

(ii) Sketch the Nyquist plot for the system, and use the Nyquist criterion to find the range of K for closed loop stability.

(iii) Briefly describe how the gain margin and phase margin estimated from the Bode plot can be obtained from the Nyquist sketch.

4. Consider the following motor control system:



(i) Find the characteristic equation of the closed loop system, and find the amplifier gain K_1 so that the natural frequency of the system is $\omega_n = 6$.

(ii) Suppose that $K_2 = 1$. Sketch the root locus of the closed loop system poles for $0 < K_1 < \infty$. (Hint: You must put the characteristic equation in the form $1 + K_1 \frac{N(s)}{D(s)} = 0$.)

(iii) For what value of K_1 will the closed loop system exhibit a stable, underdamped response with damping ratio of 0.707?

5. A system described by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

with

$$A = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

is under feedback control of the form $u = -Kx + r$ where r is the reference input.

(i) Show that (A, C) is observable.

(ii) Compute a K of the form $K = [1 \; K_2]$ so that $(A - BK, C)$ is unobservable. (I.e., the closed loop system is unobservable.)

(iii) Find the transfer function of the open loop system as well as the transfer function of the closed loop system, and compare the transfer functions to determine what the unobservability is due to.