Digital Signal Processing, 98-Elec-B2

National Exams, May 2003

- This is a closed-book, closed-notes exam. Discrete Fourier-transform and Z-transform properties tables are provided.
- Any Casio or Sharp Calculator Models are permitted.
- Internet or computers should not be accessed during the duration of the exam.
- No questions will be answered by the instructor administering the exam. Any assumptions made should be clearly stated in your answer sheet.
- The duration of the exam is 3 hours.
- There are six questions, and any **five** questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked. All questions carry equal marks.

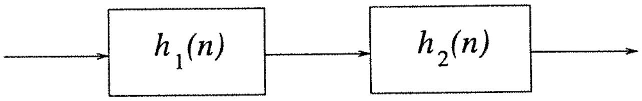

Fig. 1

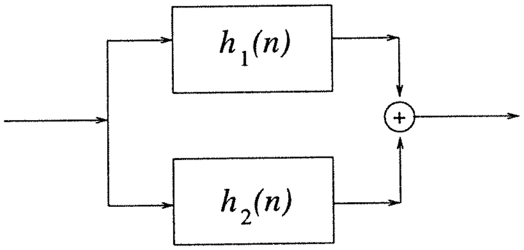

Fig. 2

Question 3 (6+4 Marks)

1. The coefficients of an FIR filter are $\{\underset{\uparrow}{-0.5}\ \ 0.5\ \ 1\ \cdot\ 0.5\ \ -0.5\}$. Determine the frequency response of the filter, and draw an optimal filter structure.
2. A system corrupts any input $x(n)$ in such a way that $x(n)$ changes to $y(n)$, where $y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2)$. Using block diagram, explain a method of removing the corruptions caused by the system.

Question 4 (5+3+2 Marks)

1. Compute the Discrete Fourier transform (DFT) of $h_1(n) = \{\underset{\uparrow}{1}\ \ 1\ \ 1\ \ 1\}$, where $\uparrow$ denotes the time index $n = 0$.
   (DFT formula $X(k) = \sum_{n=0}^{N-1} x(n)\ \exp(-j\ \frac{2\pi kn}{N})$, where $k = 0, 1, \cdots, N-1$).
2. Express the DFT of $h_2(n) = \{0\ \ 0\ \ 1\ \ 1\ \ 1\ \ 1\}$ in terms of the DFT of $h_1(n)$.
3. Explain whether $h_1(n)$ is a lowpass, highpass, or a notch filter? Give an important application of $h_1(n)$.

Question 5 (4+4+2 Marks)

An analog signal is represented as $x(t) = 2\cos(3\pi\ t)$.

1. What is the Nyquist sampling rate of the signal $x(t)$. Provide the discrete-time samples for one second duration of the signal $x(t)$.
2. Compute the FFT using decimation in time algorithm for **1 second** duration of the signal $x(t)$.
3. When compared to DFT what is the reduction in the number of additions and multiplications achieved by using FFT.

Question 6 (3+4+3 Marks)

When Bill, a novice DSP engineer was trying to record his speech signal (maximum frequency of speech = 3.3 kHz) to perform some DSP, he was noticing a periodic electrical signal interference from his wife Hilary's hair dryer machine. The periodic interference consisted of frequencies 2 kHz and 4 kHz. While recording the speech signal, Bill also noticed random noise from the nearby electric fan distorting the signal. The DSP machine that Bill was using can handle only 8 bit numbers for input. Now, help out Bill with some ideas on:

1. How to convert his speech into digital format? (Suggest him the correct sampling rate and quantization levels).
2. What type of filter(s) he has to use to remove interferences? Give him the transfer function of the filter(s).
3. Any other signal processing techniques that would make his speech analysis better.

PROPERTIES OF THE Z-TRANSFORM

Property	Time Domain	z-Domain	ROC
Notation	$x(n)$ $x_1(n)$ $x_2(n)$	$X(z)$ $X_1(z)$ $X_2(z)$	ROC: $r_2 < \|z\| < r_1$ ROC$_1$ ROC$_2$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(z) + a_2X_2(z)$	At least the intersection of and ROC$_2$
Time shifting	$x(n-k)$	$z^{-k}X(z)$	That of $X(z)$, except $z = 0$ i and $z = \infty$ if $k < 0$
Scaling in the z-domain	$a^n x(n)$	$X(a^{-1}z)$	$\|a\|r_2 < \|z\| < \|a\|r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$\frac{1}{r_1} < \|z\| < \frac{1}{r_2}$
Conjugation	$x^*(n)$	$X^*(z^*)$	ROC
Real part	$\mathrm{Re}\{x(n)\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Includes ROC
Imaginary part	$\mathrm{Im}\{x(n)\}$	$\frac{1}{2}[X(z) - X^*(z^*)]$	Includes ROC
Differentiation in the z-domain	$nx(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \|z\| < r_1$
Convolution	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$	At least, the intersection ol and ROC$_2$
Correlation	$r_{x_1x_2}(l) = x_1(l) * x_2(-l)$	$R_{x_1x_2}(z) = X_1(z)X_2(z^{-1})$	At least, the intersection ol $X_1(z)$ and $X_2(z^{-1})$
Initial value theorem	If $x(n)$ causal	$x(0) = \lim_{z\to\infty} X(z)$	
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{2\pi j}\oint_C X_1(v)X_2\left(\frac{z}{v}\right)v^{-1}dv$	At least $r_{1l}r_{2l} < \|z\| < r_{1u}r_{2u}$
Parseval's relation	$\sum_{-\infty}^{\infty} x_1(n)x_2^*(n) = \frac{1}{2\pi j}\oint_C X_1(v)X_2^*(1/v^*)v^{-1}dv$		

PROPERTIES OF THE DFT

Property	Time Domain	Frequency Domain
Notation	$x(n), y(n)$	$X(k), Y(k)$
Periodicity	$x(n) = x(n+N)$	$X(k) = X(k+N)$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(k) + a_2X_2(k)$
Time reversal	$x(N-n)$	$X(N-k)$
Circular time shift	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
Complex conjugate	$x^*(n)$	$X^*(N-k)$
Circular convolution	$x_1(n) \textcircled{N} x_2(n)$	$X_1(k)X_2(k)$
Circular correlation	$x(n) \textcircled{N} y^*(-n)$	$X(k)Y^*(k)$
Multiplication of two sequences	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \textcircled{N} X_2(k)$
Parseval's theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$