NATIONAL EXAMS May 2003

98-Elec-B3 Advanced Control Systems

3 hours duration

NOTES:

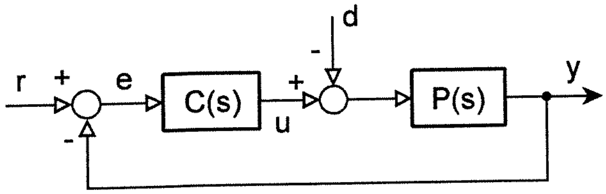
1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio FX-991 or a Sharpe EL-540. This is a closed-book examination. Tables of Laplace and z-transforms will be supplied.

3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1. Consider the following system,



$$C(s) = \frac{sK_p + K_i}{s}, \quad P(s) = \frac{1}{s(s+2)}$$

(a) Let $K_i = 0$. Find a value for K_p, say $K_p = K_{p0}$, such that the overshoot at u(t) is 10% when there is a step change at d(t). Assume r(t) = 0.

(b) Let $K_p = K_{p0}$. Find $K_{i\,max}$, the maximum value of K_i for closed loop stability.

(c) Let $K_i = K_{i\,max}/2$ and $K_p = K_{p0}$. Determine the steady state value of e(t) when r(t) = a ramp with slope 2 and d(t) = 0. Then determine the steady state value of u(t) when r(t) = 0 and d(t) = a unit step.

2. Consider the system,

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1-2\alpha \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \alpha \\ 1 \end{bmatrix}$$

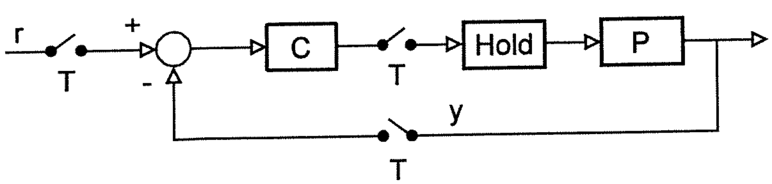
$$C = [\,1 \quad 1 \quad 0\,], \quad D = 0$$

(a) Is the system controllable? Does controllability depend on α? If so, what are the conditions for controllability? Justify your answer.

(b) Is the system observable? Does observability depend on α? If so, what are the conditions for observability? Justify your answer.

(c) Find the transfer function that relates Y(s) to U(s).

3. Consider the sampled data system with uniform sample period, T.



$$C(z) = \frac{K}{z}, \quad P(s) = \frac{1}{s+2}$$

(a) Find the discrete transfer function that relates Y(z) to R(z).

(b) What conditions on K and T result in closed loop stability? What is the condition, when in the limit T approaches zero?

4. The transfer function of a discrete time system has the form,

$$Y(z) = \frac{\alpha}{z + \beta} U(z)$$

k	y(k)	u(k)
1	5	0
2	1	0.8
3	0.6	-2
4	-0.88	0.5

Measurements of u and y are taken at time instants, k, as listed in the table.

Assuming the measurements are corrupted by zero mean white noise, find the best estimate for α and β in the least squares sense.

5. Given the system,

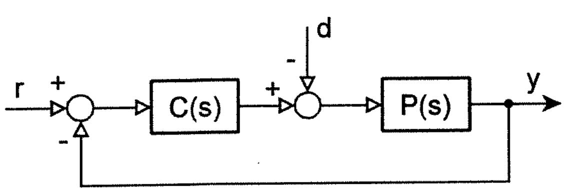
$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 0 \\ -1 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [\,1 \quad 0 \quad 1\,], \quad D = 0$$

and the observer, $\dot{x}_e(t) = Ax_e(t) + Bu(t) + K[y(t) - Cx_e(t)]$, determine the observer gain, K, such that the poles are at: -10, $-4 \pm j4$.

6. Consider the feedback system



$$C(s) = K, \quad P(s) = \frac{e^{-s}}{s(s+1)}$$

(a) Find the value of K such that the gain margin is 0.5. What is the associated phase margin?

(b) Let r(t) = 2 sin(3t) and d(t) = 1. Using the value of K found above, what is the steady state output at y?

Inverse Laplace Transforms	
F(s)	**f(t)**
$\dfrac{A}{s + \alpha}$	$Ae^{-\alpha t}$
$\dfrac{C + jD}{s + \alpha + j\beta} + \dfrac{C - jD}{s + \alpha - j\beta}$	$e^{-\alpha t}(2C\cos\beta t + 2D\sin\beta t)$
$\dfrac{A}{(s + \alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C + jD}{(s + \alpha + j\beta)^{n+1}} + \dfrac{C - jD}{(s + \alpha - j\beta)^{n+1}}$	$\dfrac{t^{n}e^{-\alpha t}}{n!}(2C\cos\beta t + 2D\sin\beta t)$

Inverse z-Transforms	
F(z)	**f(nT)**
$\dfrac{Kz}{z - a}$	Ka^{n}
$\dfrac{(C + jD)z}{z - re^{j\varphi}} + \dfrac{(C - jD)z}{z - re^{-j\varphi}}$	$2r^{n}(C\cos n\varphi - D\sin n\varphi)$
$\dfrac{Kz}{(z - a)^{r}}$ $r = 2, 3 \ldots$	$\dfrac{Kn(n - 1)\ldots(n - r + 2)}{(r - 1)!\,a^{r-1}}a^{n}$

Table of Laplace and z-Transforms		
f(t)	**F(s)**	**F(z)**
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{-\alpha T}}$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta T)}{z^2-2z\cos \beta T+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin \beta T}{z^2-2z\cos \beta T+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha T}\cos \beta T)}{z^2-2ze^{-\alpha T}\cos \beta T+e^{-2\alpha T}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha T}\sin \beta T}{z^2-2ze^{-\alpha T}\cos \beta T+e^{-2\alpha T}}$
t f(t)	$-\frac{dF(s)}{ds}$	$-zT\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha T})$