National Exams December 2005

98-BS-1, Mathematics

3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. 20 marks
2. (a) 4 marks, (b) 16 marks
3. (a) 12 marks, (b) 4 marks, (c) 4 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. (a) 3 marks, (b) 3 marks, (c) 14 marks
8. 20 marks

1. An elastic membrate in the x_1x_2-plane with boundary circle $x_1^2+x_2^2=1$ is stretched so that a point $P:(x_1,x_2)$ goes over into the point $Q:(y_1,y_2)$ given by

$$y_1 = 4x_1 + \sqrt{8}x_2,$$
$$y_2 = \sqrt{8}x_1 + 6x_2.$$

Find the principal directions of the transformation. These are the directions of the position vectors $\mathbf{x}$ of all points P for which the direction of the position vector $\mathbf{y}$ of Q is the same or exactly opposite. What shape does the boundary circle take under the deformation?

2. Consider the quadratic form $2x^2 - 6xy + 2y^2 = 13$.

   (a) What type of conic section is represented by the above quadratic form?

   (b) Transform the quadratic form to principal axes.

3. Consider the two lines defined as follows:

$$x = 3-2t, \quad y = 3, \quad z = 3-t, \text{ (parameter t)};$$
$$x = s, \quad y = 1-2s, \quad z = 2+s, \text{ (parameter s)}.$$

   (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.

   (b) Find a third line orthogonal to both lines.

   (c) Is there a plane containing both lines? If so, find an equation for that plane.

4. Find the volume of the solid region above the plane $z=-4$ and below the paraboloid $z = 4 - 2x^2 - 2y^2$.

5. Use Lagrange multipliers to find the volume of the largest box with faces parallel to the coordinate planes that can be inscribed in the ellipsoid

$$16x^2 + 4y^2 + 9z^2 = 144.$$

6. Find the general solution, $y(x)$, of the differential equation

$$2x^2y'' + xy' - y = 3x^4.$$

Note that $'$ denotes differentiation with respect to x.

7. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = 1 - 2x + y$, travelled clockwise as viewed from the positive z-axis, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$.

   (a) Evaluate the divergence of $\mathbf{v}$

   (b) Evaluate the curl of $\mathbf{v}$

   (c) Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$.

8. Solve the initial value problem

$$y'' + y' - 6y = 2e^{-3t}, \qquad y(0) = 0, \quad y'(0) = 2.$$

Note that $'$ denotes differentiation with respect to t.