# National Exams

May 2002

Applied Probability and Statistics
Ind-B1

**Notes**

1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made;

2. Candidates may use one of two calculators, the Casio or Sharp approved models. This is a **Closed Book** exam.

3. Any **five** questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are equal value.

**Q.1-** Determine whether the following statements are true or false. **[2.5 points each]**

a) Under certain mild conditions, the distribution of the sum of n samples approaches the normal as $n \to \infty$ regardless of the distribution of X.

b) Let X be the total number of success observed in 200 independent Bernoulli trials where the probability of a success is 0.03. Then we can use the Poisson approximation to binomial distribution.

c) Suppose X is a random variable with E(X) = b and Var(X) = b where b>0. Then $E(X^2) = b + b^2$.

d) Let X and Y be random variables. If the correlation of X and Y is zero then X and Y are independent.

e) Let X denotes the number of "successes" in a sample of *n* items. The variance of X is smaller when sampling "with replacement" than when sampling "without replacements"

f) Geometric distribution has memoryless property.

g) If X~Normal (2,4) and Y ~Normal (2,4) are independent random variables then (X-Y) ~Normal (0,4).

h) If the median is above the mean then this suggests that the data set may be skewed right.

**Q. 2-** Let X be a continuous variable with probability density function

$$f_X(x) \begin{cases} & \text{if} \quad -1 \le x < 0 \\ & \text{if} \quad 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of $a$. **[5 points]**

b) Find $P(X \le 0.5)$ **[5 points]**

c) Find $P(X^2 \le 0.25)$ **[5 points]**

d) Find the cumulative distribution function for X. **[5 points]**

**Q.3-** Assume that the following 10 data are obtained from a random number generator and distributed U(0,1)

0.06 0.79 0.32 0.65 0.43 0.78 0.71 0.44 0.97 0.30

**a)** Perform a Chi-square goodness-of-fit test (at $\alpha = 0.05$) to test if the 10 observations in are U(0,1). Use *three cells* designed so that the probability content of each cell is the same. Ignore any restrictions that must be considered to perform the test. **[10 points]**

**b)** Construct a P-P plot (Probability-Probability Plot). **[10 points]**

**Q.4-** The following observations are standardized yields from 24 plots divided into four blocks, each containing six similar plots. Three treatments were applied twice in each block as indicated.

| | control | Treatment F | Treatment G | Total |
|---|---|---|---|---|
| Block 1 | 0,1 | 1,-1 | 2,0 | 3 |
| Block 2 | -1,-2 | 2,1 | 1,2 | 3 |
| Block 3 | 0,2 | 1,2 | 4,3 | 12 |
| Block 4 | 3,5 | 2,4 | 7,9 | 30 |
| Total | 8 | 12 | 28 | 48 |

Consider the analysis of variance table below,

| [illegible] | SS |
|---|---|
| Blocks | 81 |
| Treatr | |
| Error | |
| Total | |

Assume an additive model with independent normally distributed errors having equal variances.

**a)** For these data, show that the sum of square errors due to treatments is equal to 28 and complete the missing cells of the Analysis of variance table. **[5 points]**

**b)** Test the significance of the treatment effects. **[5 points]**

**c)** Find an estimate of the error variance. **[5 points]**

**d)** Find a 95% confidence interval for the effect of treatment G, i.e., the difference in mean yield for treatment G compared to the control (no treatment). **[5 points]**

**Q.5-** Suppose that the number of children born in Toronto each day is a binomial random variable with mean 1000 and variance 100. Assume that the number of children born on any particular day is independent of number of children born on all other days. What is the probability that on at least one day this year, fewer than 975 children will be born in Toronto? **[20 points]**

**Q.6-** Let W denote the amount of weight (in thousands of pounds) that a certain bridge can withstand without structural damage. Civil engineers believe that W is normally distributed with mean 400 and standard deviation 40. Suppose that weights (in thousands of pounds) of a randomly selected car is a random variable with mean 3 and standard deviation 0.3.

**a)** What is the probability that the bridge can withstand 310,000 pounds without structural damage? **[5 points]**

**b)** What is the probability that 100 randomly selected cars will have a combined weight more than 310,000? **[5 points]**

**c)** What is the probability that 100 randomly selected cars will weigh enough to cause structural damage? **[5 points]**

**d)** How many cars would have to be on the bridge for the probability of structural damage to exceed 0.10? **[5 points]**

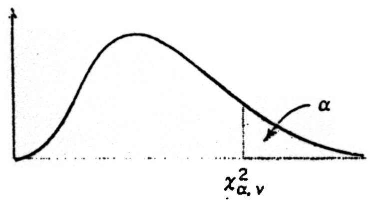
**Q.7-** The joint density function of X and Y is

$$f(x,y)=\begin{cases} 2 & \text{if} \quad 0<x<y, 0<y<1 \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

**a)** Compute the density function $f_Y(y)$ of Y. **[6 points]**

**b)** Compute the conditional density function $f_{X/Y}(x/y)$ of X, given Y=y. **[7 points]**

**c)** Compute the $E[X \mid Y=y]$. **[7 points]**



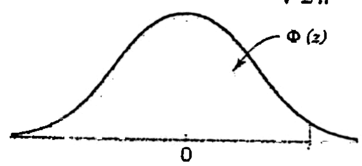

Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Square Distribution

| $\nu$ \ $\alpha$ | .995 | .990 | .975 | .950 | .900 | .500 | .100 | .050 | .025 | .010 | .005 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .00+ | .00+ | .00+ | .00+ | .02 | .45 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 |
| 2 | .01 | .02 | .05 | .10 | .21 | 1.39 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 |
| 3 | .07 | .11 | .22 | .35 | .58 | 2.37 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 |
| 4 | .21 | .30 | .48 | .71 | 1.06 | 3.36 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 |
| 5 | .41 | .55 | .83 | 1.15 | 1.61 | 4.35 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 |
|  | .68 | .87 | 1.24 | 1.64 | 2.20 | 5.35 | 10.65 | 12.59 | 14.45 | 16.81 | 18.55 |
|  | .99 | 1.24 | 1.69 | 2.17 | 2.83 | 6.35 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 7.34 | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 8.34 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 9.34 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 10.34 | 17.28 | 19.68 | 21.92 | 24.72 | 26.76 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 11.34 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 12.34 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 13.34 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 15 | 4.60 | 5.23 | 6.27 | 7.26 | 8.55 | 14.34 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 15.34 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 16.34 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.87 | 17.34 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 18.34 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 20 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 19.34 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 21 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 20.34 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 21.34 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 23 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 22.34 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 23.34 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 24.34 | 34.28 | 37.65 | 40.65 | 44.31 | 46.93 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 25.34 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 |
| 27 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 26.34 | 36.74 | 40.11 | 43.19 | 46.96 | 49.65 |
| 28 | 12.46 | 13.57 | 15.31 | 16.93 | 18.94 | 27.34 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 |
| 29 | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 28.34 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 29.34 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 39.34 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 49.33 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 59.33 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 69.33 | 85.53 | 90.53 | 95.02 | 100.42 | 104.22 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 79.33 | 96.58 | 101.88 | 106.63 | 112.33 | 116.32 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 89.33 | 107.57 | 113.14 | 118.14 | 124.12 | 128.30 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 99.33 | 118.50 | 124.34 | 129.56 | 135.81 | 140.17 |

$\nu$ = degrees of freedom.

Cumulative Standard Normal Distribution

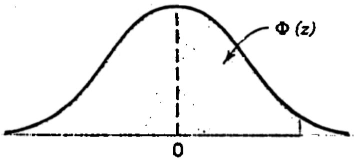
$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}}\, du$$




| z | .00 | .01 | .02 | .03 | .04 | z |
|---|---|---|---|---|---|---|
| .0 | .500 00 | .503 99 | .507 98 | .511 97 | | 0 |
| .1 | .539 83 | .543 79 | .547 76 | .551 72 | | .1 |
| .2 | .579 26 | .583 17 | .587 06 | .590 95 | | .2 |
| .3 | .617 91 | .621 72 | .625 51 | .629 30 | | .3 |
| .4 | .655 42 | .659 10 | .662 76 | .666 40 | | .4 |
| .5 | .691 46 | .694 97 | .698 47 | .701 94 | | .5 |
| .6 | .725 75 | .729 07 | .732 37 | .735 65 | | .6 |
| .7 | .758 03 | .761 15 | .764 24 | .767 30 | | .7 |
| .8 | .788 14 | .791 03 | .793 89 | .796 73 | | .8 |
| .9 | .815 94 | .818 59 | .821 21 | .823 81 | | .9 |
| 1.0 | .841 34 | .843 75 | .846 13 | .848 49 | | 1.0 |
| 1.1 | .864 33 | .866 50 | .868 64 | .870 76 | | 1.1 |
| 1.2 | .884 93 | .886 86 | .888 77 | .890 65 | | 1.2 |
| 1.3 | .903 20 | .904 90 | .906 58 | .908 24 | | 1.3 |
| 1.4 | .919 24 | .920 73 | .922 19 | .923 64 | | 1.4 |
| 1.5 | .933 19 | .934 48 | .935 74 | .936 99 | | 1.5 |
| 1.6 | .945 20 | .946 30 | .947 38 | .948 45 | | 1.6 |
| 1.7 | .955 43 | .956 37 | .957 28 | .958 18 | | 1.7 |
| 1.8 | .964 07 | .964 85 | .965 62 | .966 37 | | 1.8 |
| 1.9 | .971 28 | .971 93 | .972 57 | .973 20 | | 1.9 |
| 2.0 | .977 25 | .977 78 | .978 31 | .978 82 | | 2.0 |
| 2.1 | .982 14 | .982 57 | .983 00 | .983 41 | | 2.1 |
| 2.2 | .986 10 | .986 45 | .986 79 | .987 13 | | 2.2 |
| 2.3 | .989 28 | .989 56 | .989 83 | .990 10 | | 2.3 |
| 2.4 | .991 80 | .992 02 | .992 24 | .992 45 | | 2.4 |
| 2.5 | .993 79 | .993 96 | .994 13 | .994 30 | | 2.5 |
| 2.6 | .995 34 | .995 47 | .995 60 | .995 73 | | 2.6 |
| 2.7 | .996 53 | .996 64 | .996 74 | .996 83 | | 2.7 |
| 2.8 | .997 44 | .997 52 | .997 60 | .997 67 | | 2.8 |
| 2.9 | .998 13 | .998 19 | .998 25 | .998 31 | | 2.9 |
| 3.0 | .998 65 | .998 69 | .998 74 | .998 78 | | 3.0 |
| 3.1 | .999 03 | .999 06 | .999 10 | .999 13 | | 3.1 |
| 3.2 | .999 31 | .999 34 | .999 36 | .999 38 | | 3.2 |
| 3.3 | .999 52 | .999 53 | .999 55 | .999 57 | | 3.3 |
| 3.4 | .999 66 | .999 68 | .999 69 | .999 70 | | 3.4 |
| 3.5 | .999 77 | .999 78 | .999 78 | .999 79 | | 3.5 |
| 3.6 | .999 84 | .999 85 | .999 85 | .999 86 | | 3.6 |
| 3.7 | .999 89 | .999 90 | .999 90 | .999 90 | | 3.7 |
| 3.8 | .999 93 | .999 93 | .999 93 | .999 94 | | 3.8 |
| 3.9 | .999 95 | .999 95 | .999 96 | .999 96 | | 3.9 |

Cumulative Standard Normal Distribution (*continued*)

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$




| z | .05 | .06 | .07 | .08 | .09 | z |
|---|---|---|---|---|---|---|
| .0 | .519 94 | .523 92 | .527 90 | .531 88 | .535 86 | .0 |
| .1 | .559 62 | .563 56 | .567 49 | .571 42 | .575 34 | .1 |
| .2 | .598 71 | .602 57 | .606 42 | .610 26 | .614 09 | .2 |
| .3 | .636 83 | .640 58 | .644 31 | .648 03 | .651 73 | .3 |
| .4 | .673 64 | .677 24 | .680 82 | .684 38 | .687 93 | .4 |
| .5 | .708 84 | .712 26 | .715 66 | .719 04 | .722 40 | .5 |
| .6 | .742 15 | .745 37 | .748 57 | .751 75 | .754 90 | .6 |
| .7 | .773 37 | .776 37 | .779 35 | .782 30 | .785 23 | .7 |
| .8 | .802 34 | .805 10 | .807 85 | .810 57 | .813 27 | .8 |
| .9 | .828 94 | .831 47 | .833 97 | .836 46 | .838 91 | .9 |
| 1.0 | .853 14 | .855 43 | .857 69 | .859 93 | .862 14 | 1.0 |
| 1.1 | .874 93 | .876 97 | .879 00 | .881 00 | .882 97 | 1.1 |
| 1.2 | .894 35 | .896 16 | .897 96 | .899 73 | .901 47 | 1.2 |
| 1.3 | .911 49 | .913 08 | .914 65 | .916 21 | .917 73 | 1.3 |
| 1.4 | .926 47 | .927 85 | .929 22 | .930 56 | .931 89 | 1.4 |
| 1.5 | .939 43 | .940 62 | .941 79 | .942 95 | .944 08 | 1.5 |
| 1.6 | .950 53 | .951 54 | .952 54 | .953 52 | .954 48 | 1.6 |
| 1.7 | .959 94 | .960 80 | .961 64 | .962 46 | .963 27 | 1.7 |
| 1.8 | .967 84 | .968 56 | .969 26 | .969 95 | .970 62 | 1.8 |
| 1.9 | .974 41 | .975 00 | .975 58 | .976 15 | .976 70 | 1.9 |
| 2.0 | .979 82 | .980 30 | .980 77 | .981 24 | .981 69 | 2.0 |
| 2.1 | .984 22 | .984 61 | .985 00 | .985 37 | .985 74 | 2.1 |
| 2.2 | .987 78 | .988 09 | .988 40 | .988 70 | .988 99 | 2.2 |
| 2.3 | .990 61 | .990 86 | .991 11 | .991 34 | .991 58 | 2.3 |
| 2.4 | .992 86 | .993 05 | .993 24 | .993 43 | .993 61 | 2.4 |
| 2.5 | .994 61 | .994 77 | .994 92 | .995 06 | .995 20 | 2.5 |
| 2.6 | .995 98 | .996 09 | .996 21 | .996 32 | .996 43 | 2.6 |
| 2.7 | .997 02 | .997 11 | .997 20 | .997 28 | .997 36 | 2.7 |
| 2.8 | .997 81 | .997 88 | .997 95 | .998 01 | .998 07 | 2.8 |
| 2.9 | .998 41 | .998 46 | .998 51 | .998 56 | .998 61 | 2.9 |
| 3.0 | .998 86 | .998 89 | .998 93 | .998 97 | .999 00 | 3.0 |
| 3.1 | .999 18 | .999 21 | .999 24 | .999 26 | .999 29 | 3.1 |
| 3.2 | .999 42 | .999 44 | .999 46 | .999 48 | .999 50 | 3.2 |
| 3.3 | .999 60 | .999 61 | .999 62 | .999 64 | .999 65 | 3.3 |
| 3.4 | .999 72 | .999 73 | .999 74 | .999 75 | .999 76 | 3.4 |
| 3.5 | .999 81 | .999 81 | .999 82 | .999 83 | .999 83 | 3.5 |
| 3.6 | .999 87 | .999 87 | .999 88 | .999 88 | .999 89 | 3.6 |
| 3.7 | .999 91 | .999 92 | .999 92 | .999 92 | .999 92 | 3.7 |
| 3.8 | .999 94 | .999 94 | .999 95 | .999 95 | .999 95 | 3.8 |
| 3.9 | .999 96 | .999 96 | .999 96 | .999 97 | .999 97 | 3.9 |

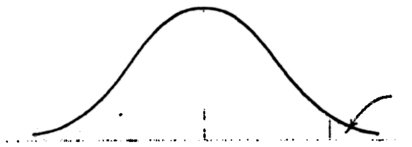



Table IV Percentage Points $t_{\alpha,\nu}$ of the $t$-Distribution

| $\nu$ \ $\alpha$ | .40 | .25 | .10 | .05 | .025 | .01 | .005 | .0025 | .001 | .0005 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1.000 | 3.078 | 6.314 | | 31.821 | 63.657 | | 318.31 | 636.62 |
| | | .816 | 1.886 | 2.920 | | 6.965 | 9.925 | | 23.326 | 31.598 |
| | | .765 | 1.638 | 2.353 | | 4.541 | 5.841 | | 10.213 | 12.924 |
| | | .741 | 1.533 | 2.132 | | 3.747 | 4.604 | | 7.173 | 8.610 |
| | | .727 | 1.476 | 2.015 | | 3.365 | 4.032 | | 5.893 | 6.869 |
| 6 | | .718 | 1.440 | 1.943 | | 3.143 | 3.707 | | 5.208 | 5.959 |
| | | .711 | 1.415 | 1.895 | | 2.998 | 3.499 | | 4.785 | 5.408 |
| 8 | | .706 | 1.397 | 1.860 | | 2.896 | 3.355 | | 4.501 | 5.041 |
| 9 | | .703 | 1.383 | 1.833 | | 2.821 | 3.250 | | 4.297 | 4.781 |
| 10 | | .700 | 1.372 | 1.812 | | 2.764 | 3.169 | | 4.144 | 4.587 |
| 11 | | .697 | 1.363 | 1.796 | | 2.718 | 3.106 | | 4.025 | 4.437 |
| 12 | | .695 | 1.356 | 1.782 | | 2.681 | 3.055 | | 3.930 | 4.318 |
| 13 | | .694 | 1.350 | 1.771 | | 2.650 | 3.012 | | 3.852 | 4.221 |
| 14 | | .692 | 1.345 | 1.761 | | 2.624 | 2.977 | | 3.787 | 4.140 |
| 15 | | .691 | 1.341 | 1.753 | | 2.602 | 2.947 | | 3.733 | 4.073 |
| 16 | | .690 | 1.337 | 1.746 | | 2.583 | 2.921 | | 3.686 | 4.015 |
| 17 | | .689 | 1.333 | 1.740 | | 2.567 | 2.898 | | 3.646 | 3.965 |
| 18 | | .688 | 1.330 | 1.734 | | 2.552 | 2.878 | | 3.610 | 3.922 |
| 19 | | .688 | 1.328 | 1.729 | | 2.539 | 2.861 | | 3.579 | 3.883 |
| 20 | | .687 | 1.325 | 1.725 | | 2.528 | 2.845 | | 3.552 | 3.850 |
| 21 | | .686 | 1.323 | 1.721 | | 2.518 | 2.831 | | 3.527 | 3.819 |
| 22 | | .686 | 1.321 | 1.717 | | 2.508 | 2.819 | | 3.505 | 3.792 |
| 23 | | .685 | 1.319 | 1.714 | | 2.500 | 2.807 | | 3.485 | 3.767 |
| 24 | | .685 | 1.318 | 1.711 | | 2.492 | 2.797 | | 3.467 | 3.745 |
| 25 | | .684 | 1.316 | 1.708 | | 2.485 | 2.787 | | 3.450 | 3.725 |
| 26 | | .684 | 1.315 | 1.706 | | 2.479 | 2.779 | | 3.435 | 3.707 |
| 27 | | .684 | 1.314 | 1.703 | | 2.473 | 2.771 | | 3.421 | 3.690 |
| 28 | | .683 | 1.313 | 1.701 | | 2.467 | 2.763 | | 3.408 | 3.674 |
| 29 | | .683 | 1.311 | 1.699 | | 2.462 | 2.756 | | 3.396 | 3.659 |
| 30 | | .683 | 1.310 | 1.697 | | 2.457 | 2.750 | | 3.385 | 3.646 |
| 40 | | .681 | 1.303 | 1.684 | | 2.423 | 2.704 | | 3.307 | 3.551 |
| 60 | | .679 | 1.296 | 1.671 | | 2.390 | 2.660 | | 3.232 | 3.460 |
| 120 | | .677 | 1.289 | 1.658 | | 2.358 | 2.617 | | 3.160 | 3.373 |
| ∞ | | .674 | 1.282 | 1.645 | | 2.326 | 2.576 | | 3.090 | 3.291 |

$\nu$ = degrees of freedom.

Percentage Points of the *F*-Distribution (*continued*)

$f_{0.01,\nu_1,\nu_2}$

| Degrees of freedom for the denominator ($\nu_2$) \ Degrees of freedom for the numerator ($\nu_1$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4052 | 4999.5 | 5403 | 5625 | 5764 | 5859 | 5928 | 5982 | 6022 | 6056 | 6106 | 6157 | 6209 | 6235 | 6261 | 6287 | 6313 | 6339 | 6366 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.87 | 26.69 | 26.00 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.20 | 9.11 | 9.02 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.14 | 7.06 | 6.97 | 6.88 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.82 | 5.74 | 5.65 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.03 | 4.95 | 4.46 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.48 | 4.40 | 4.31 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.17 | 4.08 | 4.00 | 3.91 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.25 | 4.10 | 4.02 | 3.94 | 3.86 | 3.78 | 3.69 | 3.60 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.62 | 3.54 | 3.45 | 3.36 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.82 | 3.66 | 3.59 | 3.51 | 3.43 | 3.34 | 3.25 | 3.17 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.66 | 3.51 | 3.43 | 3.35 | 3.27 | 3.18 | 3.09 | 3.00 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.36 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.13 | 3.05 | 2.96 | 2.87 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.41 | 3.26 | 3.18 | 3.10 | 3.02 | 2.93 | 2.84 | 2.75 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.31 | 3.16 | 3.08 | 3.00 | 2.92 | 2.83 | 2.75 | 2.65 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.23 | 3.08 | 3.00 | 2.92 | 2.84 | 2.75 | 2.66 | 2.57 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.15 | 3.00 | 2.92 | 2.84 | 2.76 | 2.67 | 2.58 | 2.59 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.69 | 2.61 | 2.52 | 2.42 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.17 | 3.03 | 2.88 | 2.80 | 2.72 | 2.64 | 2.55 | 2.46 | 2.36 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.12 | 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.50 | 2.40 | 2.31 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.07 | 2.93 | 2.78 | 2.70 | 2.62 | 2.54 | 2.45 | 2.35 | 2.26 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.40 | 2.31 | 2.21 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.99 | 2.85 | 2.70 | 2.62 | 2.54 | 2.45 | 2.36 | 2.27 | 2.17 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.96 | 2.81 | 2.66 | 2.58 | 2.50 | 2.42 | 2.33 | 2.23 | 2.13 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.93 | 2.78 | 2.63 | 2.55 | 2.47 | 2.38 | 2.29 | 2.20 | 2.10 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.90 | 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.26 | 2.17 | 2.06 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.87 | 2.73 | 2.57 | 2.49 | 2.41 | 2.33 | 2.23 | 2.14 | 2.03 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.21 | 2.11 | 2.01 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.66 | 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.02 | 1.92 | 1.80 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.84 | 1.73 | 1.60 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.66 | 1.53 | 1.38 |
| ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.47 | 1.32 | 1.00 |