2002 N tional Exams

98-Mec-B4

YSTEM ANALYSIS AND CONTROLS

Ho D rati

NOTES

1. If dou exists to the interpretation f any question, the candidate urged to submit ith the answer paper statement f any assumpti made
2. Candidates may one of tw calculators, Casi FX 99 <u>or</u> Sharp EL 540 Thi is <u>Closed Book</u> exam. No aids other than semi-log graph papers are permitted.

   Any fo questions constitute complete paper Only the first four questi they appear in yo answer book will be marked.

4. Al questi are f equal value

1. a) Determine if the following characteristic equation represents a stable system:

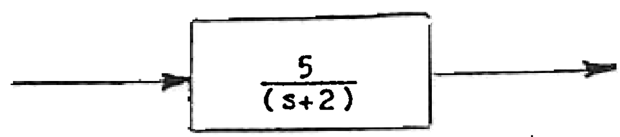
$$s^3 + 4s^2 + 8s + 12 = 0$$

b) The characteristic equation of a given system is:

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

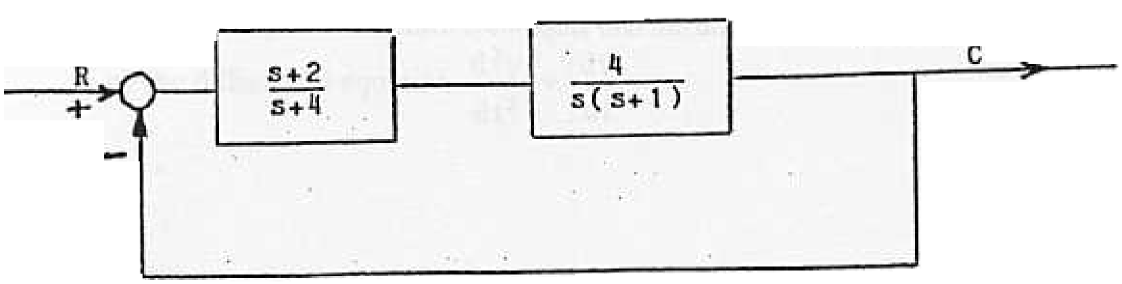
What restrictions must be placed upon the parameter K in order to insure that the system is stable?

c) The block diagram below depicts a first order lag system



Derive an expression for the output if the input, (at time t = 0), is a unit ramp. How would you obtain the response to a unit impulse from the expression which you have derived.

2. For the stable system



a) Detemine the system type.

b) Find the steady-state error for a unit step input, a unit ramp input, and a unit parabolic input.

3. a) Construct the root-locus for $K > 0$ for the transfer function

$$= \frac{K}{s(s+1)(s^2+7s+12)}$$

b) If the design value for the gain is $K = 6$, calculate gain margin.

c) Determine the value of the gain factor K for which the system with the above open loop transfer function has closed loop poles with a damping ratio

$\zeta = 0.5$.

4. a) Using the Laplace transform technique, find the transient and steady-state responses of the system described by the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 1$ with initial conditions $y(0^+)$ and $\left.\frac{dy}{dt}\right|_{t=0^+} = 1$.

b) Using the Laplace transform technique, find the unit impulse response of the system described by the differential equation $\frac{d^3y}{dt^3} + \frac{dy}{dt} = x$.

5. The block diagram of a control system is shown in Fig. 1.

a) When $k_i = 10$, r(t) is a unit step function and d(t) = 0. Obtain the value of the steady-state error.

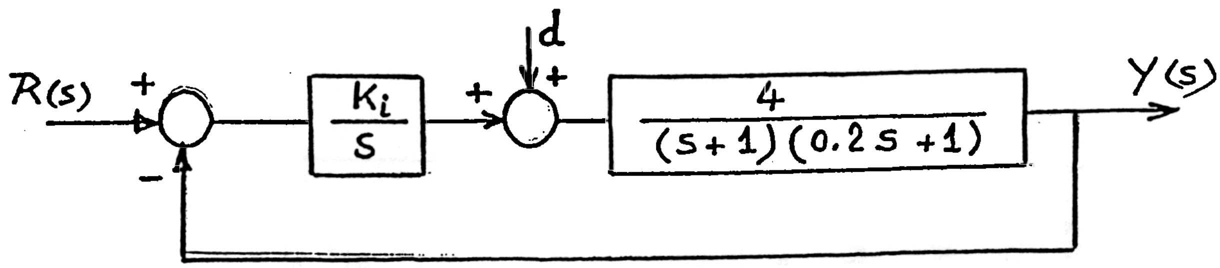


Fig. Block diagram of a control system

b) When the input is a ramp such that r(t) = t, d = 0, it is desired to limit the steady-state error to a value equal to or less than 0.2. Obtain the value of k_i and determine if this requirement is consistent with the requirement of stability.

TABLE 6-1

Laplace Transform Pairs

Functions of Time: $f(t)$ for $0 \le t$	Laplace Transform: $\mathcal{L}[f(t)]$
1. $f(t)$	$\int_0^\infty f(t)e^{-st}\,dt = F(s)$
2. $x(t) + y(t)$	$X(s) + Y(s)$
3. $Kf(t)$	$KF(s)$
4. $\frac{df(t)}{dt}$	$sF(s) - f(0)$
5. $\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$
6. $\frac{d^nf(t)}{dt^n}$	$s^nF(s) - \sum_{i=1}^{n} s^{n-i}\frac{d^{i-1}f(0)}{dt^{i-1}}$
7. $\int_0^t f(t)\,dt$	$\frac{1}{s}F(s)$
8. 1 or $u(t)$	$\frac{1}{s}$
9. t	$\frac{1}{s^2}$
10. t^n for $n > -1$	$\frac{n!}{s^{n+1}}$
11. e^{-at}	$\frac{1}{s+a}$
12. te^{-at}	$\frac{1}{(s+a)^2}$
13. t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
14. $1 - e^{-at}$	$\frac{a}{s(s+a)}$
15. $e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
16. $ae^{-at} - be^{-bt}$	$\frac{(a-b)s}{(s+a)(s+b)}$
17. $\sin \alpha t$	$\frac{\alpha}{s^2+\alpha^2}$
18. $\cos \alpha t$	$\frac{s}{s^2+\alpha^2}$

TABLE 6-1 (*Continued*)

Functions of Time: $f(t)$ for $0 \le t$	Laplace Transform: $\mathcal{L}[f(t)]$
19. $t \sin \alpha t$	$\frac{2\alpha s}{(s^2+\alpha^2)^2}$
20. $t \cos \alpha t$	$\frac{s^2-\alpha^2}{(s^2+\alpha^2)^2}$
21. $e^{-at}\sin \alpha t$	$\frac{\alpha}{(s+a)^2+\alpha^2}$
22. $e^{-at}\cos \alpha t$	$\frac{s+a}{(s+a)^2+\alpha^2}$
23. $e^{-\zeta\omega_n t}\sin \omega_n(1-\zeta^2)^{1/2}t$ for $\zeta < 1$	$\frac{\omega_n(1-\zeta^2)^{1/2}}{s^2+2\zeta\omega_n s+\omega_n^2}$
24. $e^{-\zeta\omega_n t}\sinh \omega_n(\zeta^2-1)^{1/2}t$ for $\zeta > 1$	$\frac{\omega_n(\zeta^2-1)^{1/2}}{s^2+2\zeta\omega_n s+\omega_n^2}$
25. $1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$, $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$, for $\zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
26. $\begin{cases} f(t-a) & \text{where } t > a \\ 0 & \text{where } t < a \end{cases}$	$e^{-as}F(s)$