PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – December 2002

98-Mec-B9 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) that he/she has had made with the answer.

2. The examination paper is open book and so candidates are permitted to make use of any textbooks, references or notes that they wish to use.

3. Any non-communicating calculator is permitted. A calculator that can handle small matrices will speed the solving of the problems. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.

4. Candidates are required to attempt five questions. Solve all problems using finite element method.

5. All questions carry the same value. Indicate which five questions are to be marked on the cover of the first examination workbook.

PROBLEM 1 (20 POINTS)

Consider the following differential equation:

$$\frac{d^2u}{dx^2} = 6x + 2 \qquad \text{for} \quad 0 \le x \le 1$$

that has the boundary conditions:

$$u(0) = 0 \quad ; \quad u(1) = 1$$

a. Comment on the boundary conditions.

b. Using the trial function:

$$\phi_j = x^j (1 - x)$$

find a two parameter (N=2) Galerkin solution.

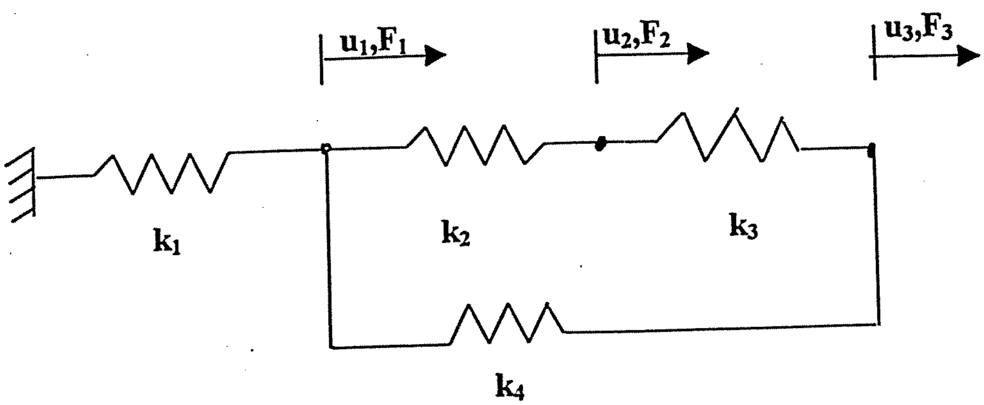
c. Compare the approximate solution found in (b) with the exact solution.

PROBLEM 2 (20 POINTS)

The structure shown below consists of linear springs whose stiffnesses are k_1, k_2, k_3, and k_4. Only horizontal displacements are allowed.

a. In a matrix form, write the three equilibrium equations of the structure. The degees of freedom are u_1, u_2 and u_3.

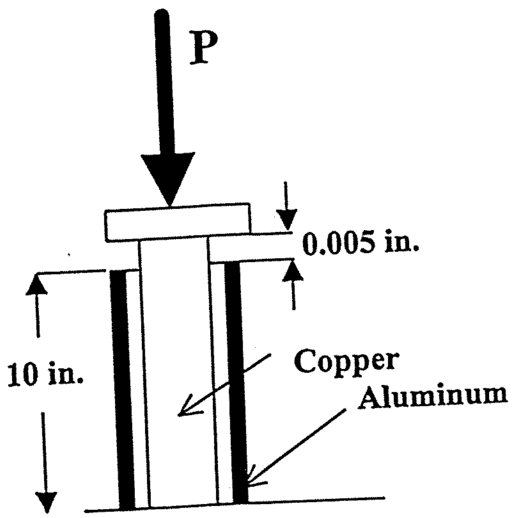
b. Let $k_1=k_2=k_3=k_4=k$ and $F_1=F_2=0$. Determine u_1, u_2 and u_3 in terms of k and F_3.



PROBLEM 3 (20 POINTS)

A copper rod is placed in an aluminum sleeve. The rod is 0.005in longer than the sleeve. Using finite element methods find the maximum safe load P that can be applied to the bearing plate, using the following data:

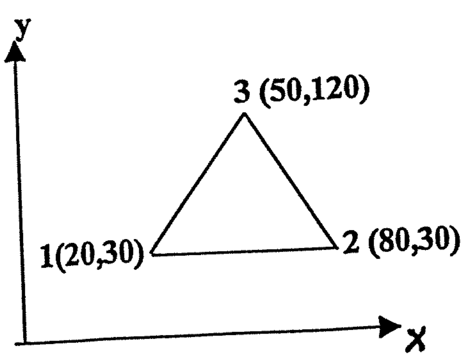
	Copper	Aluminum
Area A (in^2)	2	3
E (lb/in^2)	17×10^6	10×10^6
Allowable stress (ksi)	20	10



PROBLEM 4 (20 POINTS)

Find the stiffness matrix for the triangular element shown below. The coordinates are in millimetres. Assume plane strain conditions.

E = 200 GPA
t = 10 mm
ν = 0.3



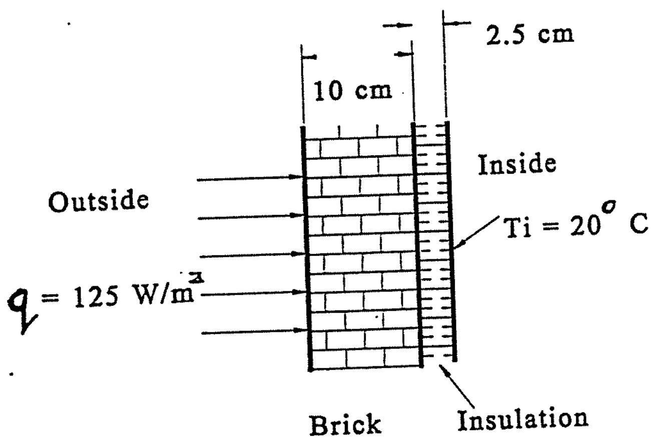
PROBLEM 5 (20 POINTS)

The exterior surface of a wall is heated by sun at a rate of $q = 125\ W/m^2$.

The wall is composed of a layer of brick and a layer of thermal insulation.
The brick is 10 cm thick, and has a conductivity of $k_{brick} = 0.5\ W/m\,^{\circ}C$.
The insulation is 2.5 cm thick and has a conductivity of $k_{insulation} = 0.04\ W/m\,^{\circ}C$.

The inside surface of the wall is maintained at $T_i = 20^{\circ}C$.

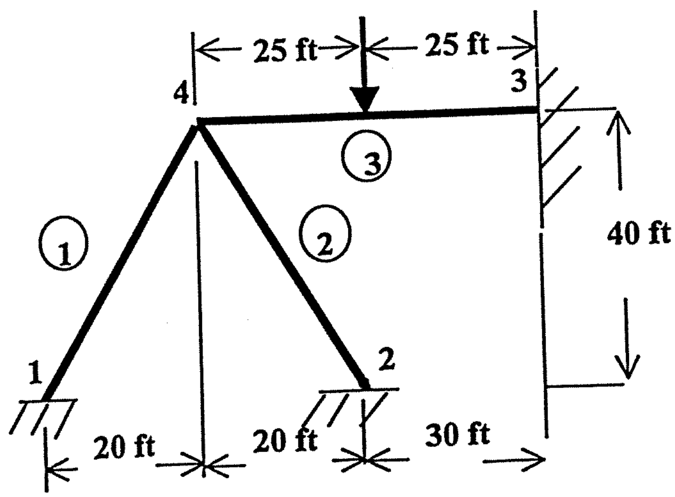
Assuming that all sun heat penetrates the wall (no heat losses from the outside surface of the wall), calculate the two element solution for the temperature distribution in the wall.



PROBLEM 6 (20 POINTS)

For the rigid frame shown below, using finite elements stiffness method, calculate:

a. **nodal displacements and rotation at node 4,**
b. **the reactions,**
c. **the forces in each element.**



For all elements:
$I = 800\ in^4$.
$E = 30 \times 10^6\ lb/in^2$
$A = 8\ in^2$

PROBLEM 7 (20 POINTS)

PART A. 10 points

Explain in a sentence or two the following concepts:

- **skyline solution**
- **symmetric banded matrix**
- **Gauss elimination**
- **to what does the term *degree of freedom* refer?**

PART B. 10 points

Model the notched bar shown below, using symmetry. Decide on the constrasints and explain your solution. No calculations required.