National Exams May 2002
98-Mec-A3, Kinematics and Dynamics of Machines
3 Hours Duration

Notes

1. If in doubt as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK exam. Candidates may use one of the two calculators, the Casio or Sharp approved models.
3. Any four questions (25 marks for each question) constitute a complete paper. If you choose to answer more than four questions, only the first four questions as they appear in your answer book will be marked.

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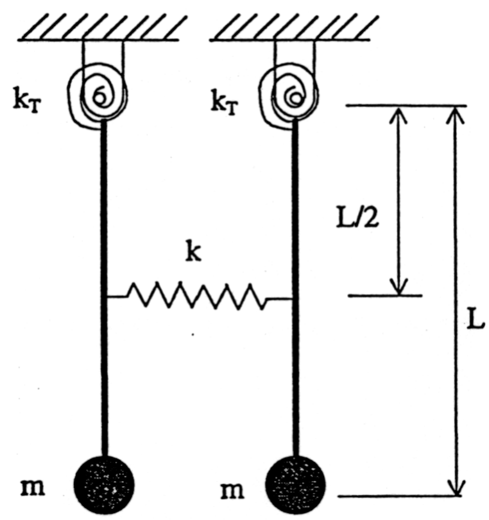
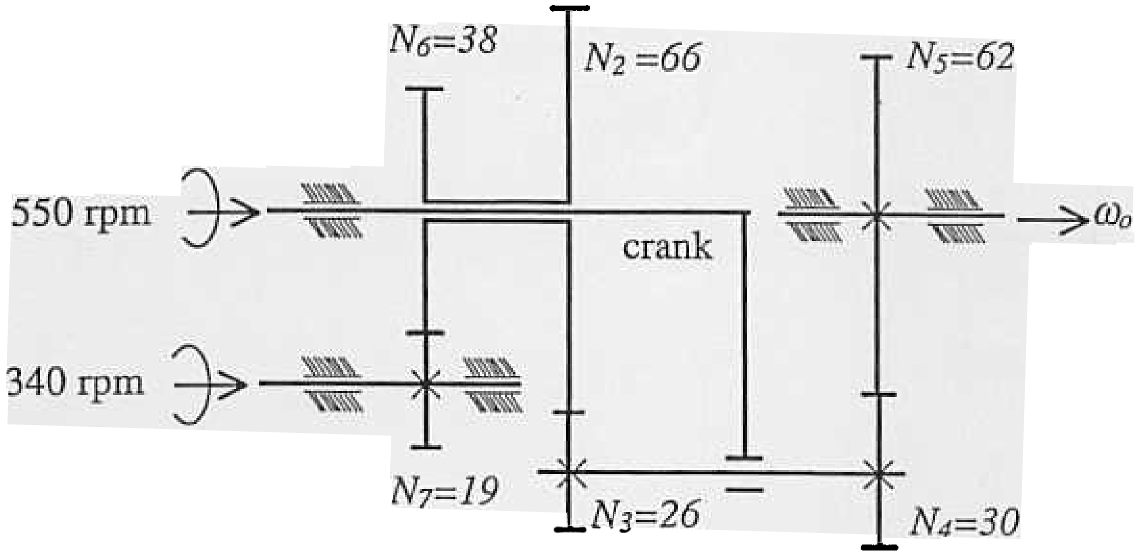
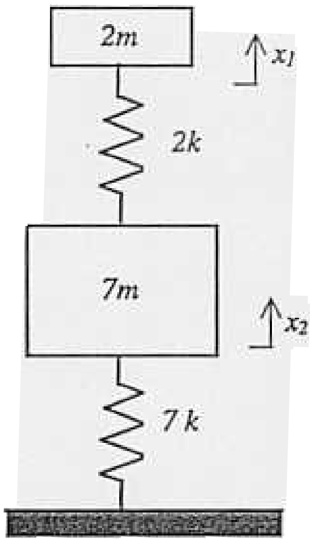
1. Each of the two massless rigid bars is hinged at one end and connected to a point mass at the other end. A torsional spring is attached at the hinged end of each bar. A linear spring is then used to couple motions of the two bars. Assume that the two torsional springs and one linear spring are undeformed when the two bars are in the vertical positions. Determine
   (a) an appropriate set of generalized coordinates
   (b) the kinetic energy in terms of the generalized coordinates
   (c) the potential energy in terms of the generalized coordinates
   (d) the Lagrangian in terms of the generalized coordinates
   (e) the equations of motion for small amplitude vibration using the Lagrange equations.



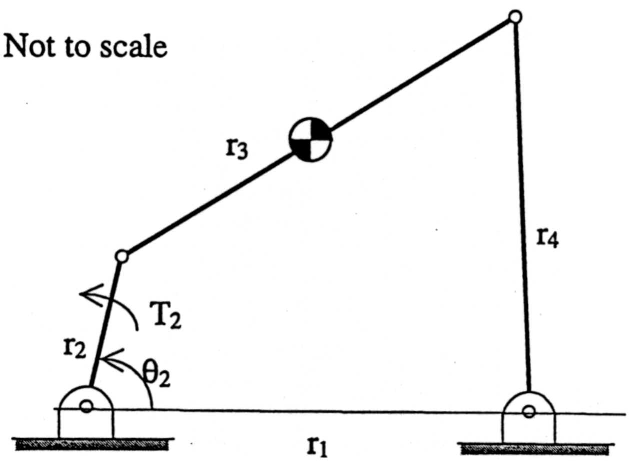
2. A gear transmission system consists of a crankshaft and three pairs of spur gears of identical diametral pitch, as shown below. Determine the output angular velocity ω_o



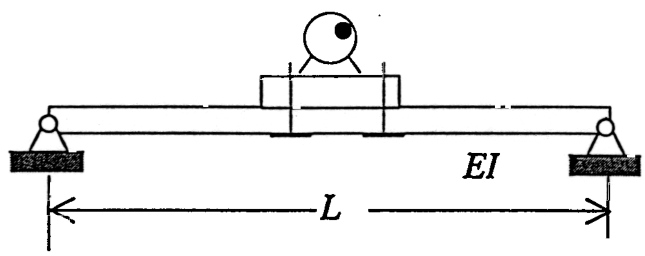
3. A coupled vibration system is shown below. For m = 5 kg and k = 10 kN/m, determine
(a) the equations of motion of the system in matrix form,
(b) the natural frequencies in Hz,
(c) the modal vectors, and
(d) the modal masses and the modal vectors normalized with respect to the mass matrix.



4. A four-bar function generation mechanism (first circuit) is shown below. The lengths of the base, input, coupler and output links are $r_1 = 98.4\,\text{mm}$, $r_2 = 33.5\,\text{mm}$, $r_3 = 100\,\text{mm}$, $r_4 = 86.2\,\text{mm}$, respectively. If the input link rotates at a constant angular velocity of 2000 rpm (CCW), determine for $\theta_2 = 78^\circ$ using the analytical method
   (a) the angular positions of the coupler and output link
   (b) the angular velocities of the coupler and output link
   (c) the angular accelerations of the coupler and output link
   (d) the linear acceleration of the midpoint (also the mass center) of the coupler
   (e) the magnitude and sense of the torque to be applied to input link from base link to overcome the inertia of link 3 ($m_3 = 0.5\,\text{kg}$, $I_{G,3} = 0.015\,\text{kg}\cdot\text{m}^2$).

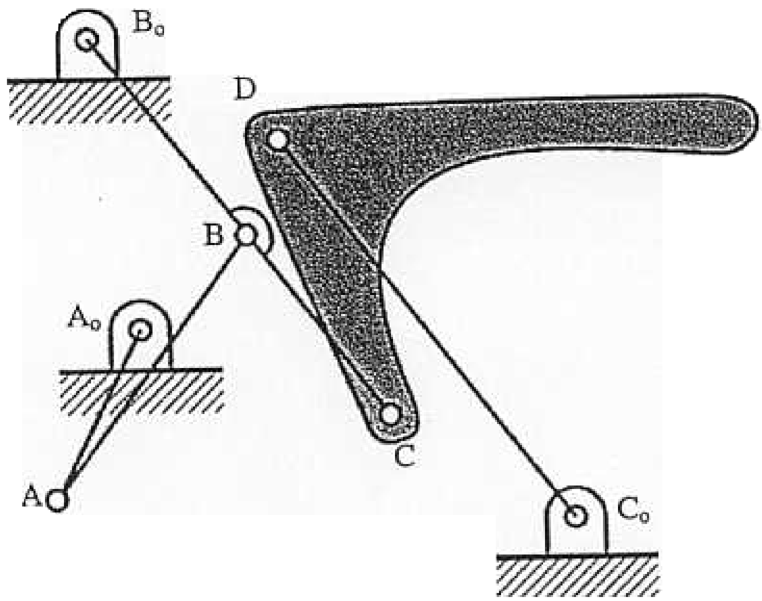


5 A motor of mass $M = 10$ kg is mounted at the midspan of a massless beam ($EI = 250\,\text{Nm}^2$, $L = 0.7$ m) as shown below. During operation, because of unbalance, the motor produces a vertical dynamic force $F(t) = me\omega^2 \sin\omega t$ N, where me, defined as the amount of unbalance is 0.005 kg m; ω is the rotational speed in rad/s. Determine
   a) the equation of motion of the motor in the vertical direction,
   b) the disastrous rotational speed of the motor, and
   c) the vertical force transmitted to the ground at each end of the beam if the motor runs at 95% of the disastrous speed calculated in (b).



6. A six-bar mechanism is given on next page. At the position shown, the input link rotates at a constant angular velocity of 20 rad/s (CCW), determine
   (a) the angular velocity C_0D,
   (b) the angular acceleration of link C_0D, and
   (c) the linear acceleration of pin D.

[Use the space provided below for velocity and acceleration diagrams]



B₀
D
B
A₀
C
A
C₀