2002 National Exams

98-Mec-B4

SYSTEM ANALYSIS AND CONTROLS

3 Hours Duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use one of two calculators, a Casio FX-991 or Sharp EL-540. This is a Closed Book exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. Draw the Bode diagram representation of the frequency response for the transfer functions given by:

a) $$GH(s) = \frac{(s + 3)}{(s^2 + 4s + 16)}$$

b) $$GH(s) = \frac{(1 + 0.5\,s)}{s^2}$$

2. Draw the root locus for the following open-loop transfer function.

$$GH(s) = \frac{K}{(s + 1)\ (s^2 + s + 1)}$$

Determine the range of the gain for which the system is stable.

3. a) A feedback control system has a characteristic equation:

$$s^3 + (4 + K)\,s^2 + 6s + 16 + 8K = 0$$

The parameter K must be positive. What is the maximum value K can assume before the system becomes unstable? When K is equal to the maximum value, the system oscillates. Determine the frequency of oscillation.

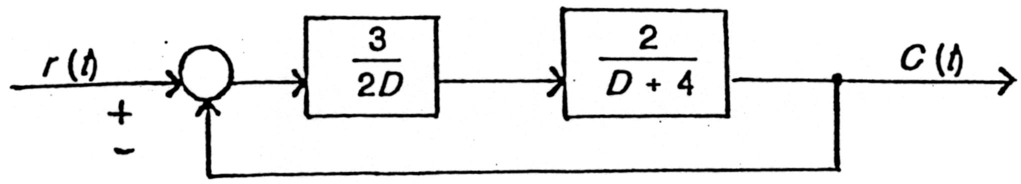
b) The dynamics of a system are described by the differential equation:

$$y(t) = \frac{10\ (2D + 1)}{(D + 2)\ (D + 5)}\ f(t)$$

where $D = \frac{d}{dt}$

Use the Laplace transform method to determine the response y(t) when all initial conditions are zero and the forcing function f(t) is a unit step function.

4. Determine the position, velocity, and acceleration error constants, and the steady-state error to a unit step, a unit ramp, and a unit parabolic input for the system shown in the figure below.



where $D = \frac{d}{dt}$

5. To improve the transient behaviour of a system, a controller with proportional and derivative action is added as shown in the figure below. Determine the value of k such that the resulting system will have a damping ratio of 0.5. What is the response c(t) of this resulting system to <u>a unit step function</u> r(t) when all initial conditions are zero?

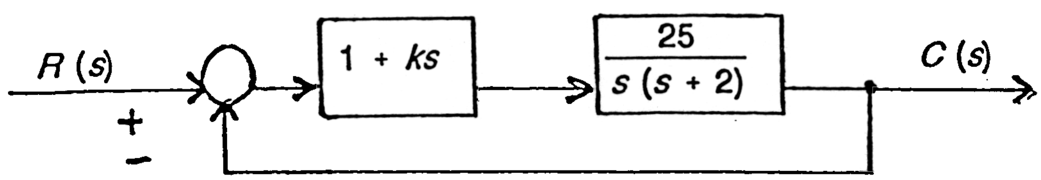


TABLE 6-1

LAPLACE TRANSFORM PAIRS

Functions of Time: $f(t)$ for $0 \le t$	Laplace Transforms: $\mathcal{L}[f(t)]$
1. $f(t)$	$\int_0^\infty f(t)e^{-st}\,dt = F(s)$
2. $x(t) + y(t)$	$X(s) + Y(s)$
3. $Kf(t)$	$KF(s)$
4. $\frac{df(t)}{dt}$	$sF(s) - f(0)$
5. $\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0) - \frac{df(0)}{dt}$
6. $\frac{d^nf(t)}{dt^n}$	$s^nF(s) - \sum_{l=1}^{n} s^{n-l}\frac{d^{l-1}f(0)}{dt^{l-1}}$
7. $\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
8. 1 or $u(t)$	$\frac{}{s}$
9. t	$\frac{1}{s^2}$
10. t^n for $n > -$	$\frac{n!}{s^{n+1}}$
11. e^{-at}	$\frac{1}{s+a}$
12. te^{-at}	$\frac{1}{(s+a)^2}$
13. t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
14. $- e^{-at}$	$\frac{a}{s(s+a)}$
15. $e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
16. $ae^{-at} - be^{-bt}$	$\frac{(a-b)s}{(s+a)(s+b)}$
17. $\sin \alpha t$	$\frac{\alpha}{s^2+\alpha^2}$
18. $\cos \alpha t$	$\frac{s}{}$

TABLE 6-1 (*Continued*)

Functions of Time: $f(t)$ for $0 \le t$	Laplace Transforms: $\mathcal{L}[f(t)]$
19. $t \sin \alpha t$	$\frac{2\alpha s}{(s^2+\alpha^2)^2}$
20. $t \cos \alpha t$	$\frac{s^2-\alpha^2}{(s^2+\alpha^2)^2}$
21. $e^{-at}\sin \alpha t$	$\frac{\alpha}{(s+a)^2+\alpha^2}$
22. $e^{-at}\cos \alpha t$	$\frac{s+a}{(s+a)^2+\alpha^2}$
23. $e^{-\zeta\omega_n t}\sin \omega_n(1-\zeta^2)^{1/2}t$ for $\zeta < 1$	$\frac{\omega_n(1-\zeta^2)^{1/2}}{s^2+2\zeta\omega_n s+\omega_n^2}$
24. $e^{-\zeta\omega_n t}\sinh \omega_n(\zeta^2-1)^{1/2}t$ for $\zeta > 1$	$\frac{\omega_n(\zeta^2-1)^{1/2}}{s^2+2\zeta\omega_n s+\omega_n^2}$
25. $1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ for $\zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
26. $\begin{cases} f(t-a) & \text{where } t > a \\ 0 & \text{where } t < a \end{cases}$	$e^{-as}F(s)$