

PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS - May 2002

98-Mec-B9 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

- 1. If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) that he/she has had made with the answer.**
- 2. The examination paper is open book and so candidates are permitted to make use of any textbooks, references or notes that they wish to use.**
- 3. Any non-communicating calculator is permitted. A calculator that can handle small matrices will speed the solving of the problems. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**
- 4. Candidates are required to attempt five questions. Solve all problems using finite element method.**
- 5. All questions carry the same value. Indicate which five questions are to be marked on the cover of the first examination workbook.**

PROBLEM 1 (20 POINTS)

Solve the following differential equation:

$$\frac{d^2 T}{dx^2} + 1000x^2 = 0 \quad \text{for } 0 \leq x \leq 1$$

subject to boundary conditions:

$$T(0) = 0 \quad \text{and,} \quad T(1) = 0$$

by using the Ritz method with the trial function

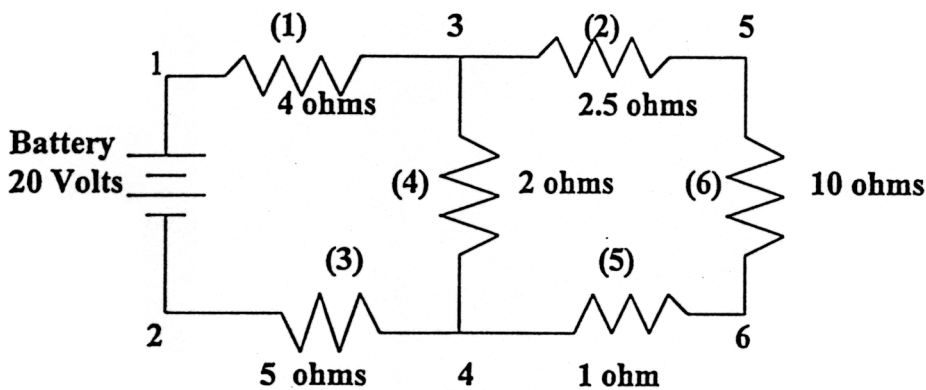
$$N_1(x) = x(1 - x^2)$$

Compare with the exact solution.

PROBLEM 2 (20 POINTS)

An electrical network with one 20 V battery and six resistors is shown below. Assume that node 1 is grounded (0 volts) and node two is at a potential of 20 V.

Using finite elements determine the voltages at all the node points and the currents through all of the resistors.



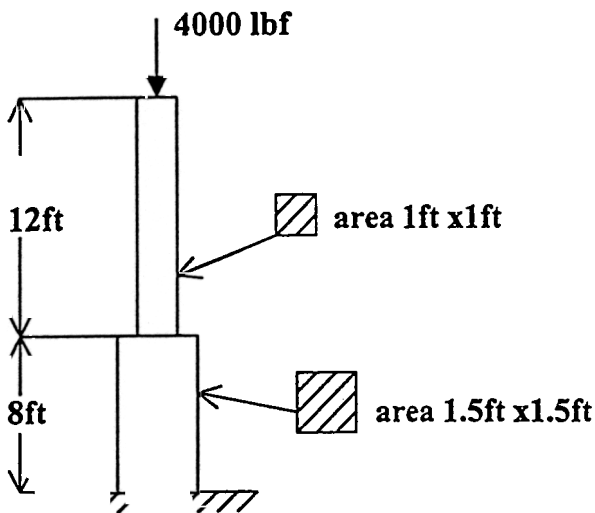
PROBLEM 3 (20 POINTS)

A two section concrete rod has square cross sections. The properties of each section are shown on the drawing below. Specific weight of concrete is $\gamma=100 \text{ lbf/ft}^3$. The rod is fixed at the bottom and subjected to an upward force of 4000 lbf at the upper end, as indicated.

Calculate the displacement of the rod due to its own weight and the upward force. Determine the stresses in each section of the rod.

Lower rod:
 Width = 1.5 ft
 $E = 4 \times 10^6 \text{ lbf/in}^2$
 $L = 8 \text{ ft}$

Upper rod:
 Width = 1 ft
 $E = 4 \times 10^6 \text{ lbf/in}^2$
 $L = 12 \text{ ft}$



PROBLEM 4 (20 POINTS)

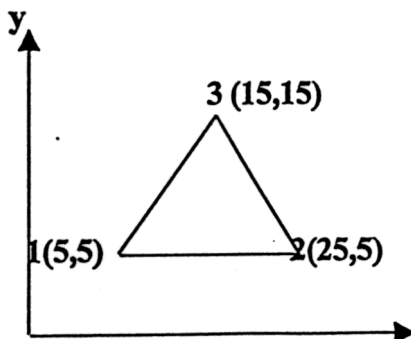
For the plane strain element shown below, the nodal displacements are given as:

$$u_1 = 0.005 \text{ mm} \quad u_2 = 0.000 \text{ mm} \quad u_3 = 0.005 \text{ mm}$$

$$v_1 = 0.002 \text{ mm} \quad v_2 = 0.000 \text{ mm} \quad v_3 = 0.000 \text{ mm}$$

Determine the element stresses σ_x , σ_y , τ_{xy} and σ_1 and σ_2 , and the principal angle θ_p .

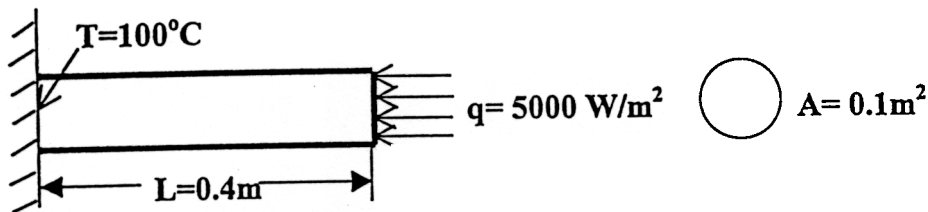
The thickness of each plate is 1 mm; $E = 70 \text{ MPa}$ and $\nu = 0.3$.



PROBLEM 5 (20 POINTS)

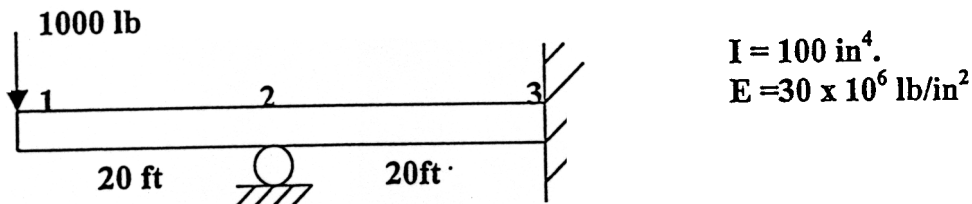
A fin shown below is insulated on the perimeter. The left end has a constant temperature of 100°C . A heat flux of 5000 W/m^2 acts on the right end of the fin. Thermal conductivity is $k=6\text{ W/m}^{\circ}\text{C}$, and the area of the fin is 0.1 m^2 . Determine the temperature at $L/4$, $L/2$, $3L/4$ and L , where $L=0.4\text{m}$.

Formulate the problem, using finite elements analysis.



PROBLEM 6 (20 POINTS)

For the beam under the loading shown in determine the displacement at the free end and the slope at the middle support.



PROBLEM 7 (20 POINTS)

PART A. 10 points

Explain in a sentence or two the following concepts:

skyline solution

symmetric banded matrix

Gauss elimination

to what does the term *degree of freedom* refer?

PART B. 10 points

b. Number the node and the elements for the truss shown below. Make use of the symmetry in such a way as to reduce the model. Draw the new model and state clearly all your assumptions. Do not make any calculations.

