

National Exams May 2003
98-Mec-A2, Fluid Mechanics and Applications

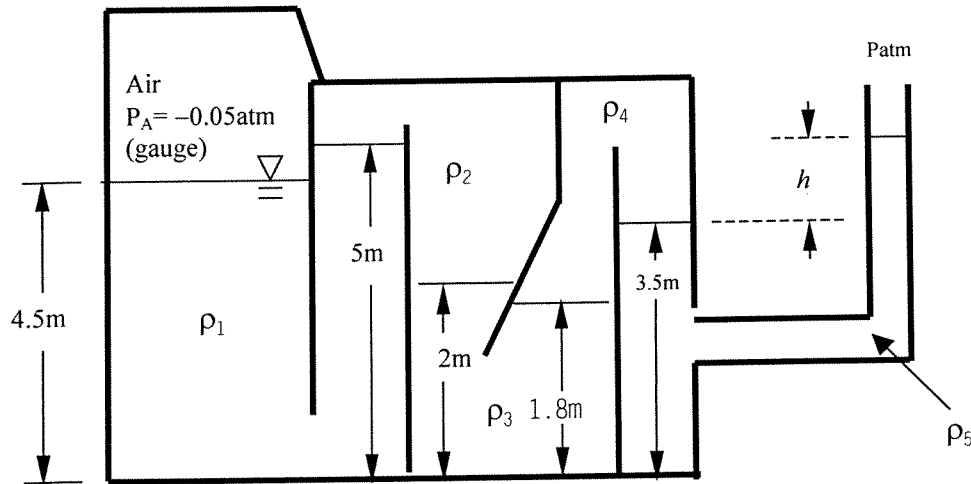
3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use a calculator of either the Casio or Sharp approved models. The exam is CLOSED BOOK. The candidate is permitted to bring into the exam and use both sides of an 8.5 by 11 inch hand-written aid sheet.
3. Answer any 3 of the 4 questions in part A and any 3 of the 5 question of part B. If more questions are attempted, these will be marked in the order they are presented.
4. Part A is weighted 40% and Part B 60% of the total exam marks. Within each section, the problems are equally weighted.

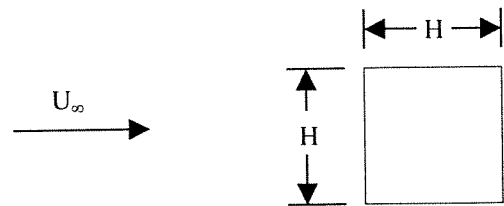
PART A: Answer any 3 of the following 4 questions.

Question A1: What is the height of the fluid column, h , as measured from the manometer tube for the conditions shown? The manometer end is open to atmosphere. The fluid densities are given as: $\rho_1=1000\text{kg/m}^3$; $\rho_2=680\text{kg/m}^3$; $\rho_3=900\text{kg/m}^3$; $\rho_4=780\text{kg/m}^3$ and $\rho_5=13600\text{kg/m}^3$.



Question A2: Vortex shedding refers to the periodic oscillations of the velocity field measured in the wake of a bluff body (a body which is not streamlined). The vortex shedding is characterised in terms of the frequency of oscillation, f . The characteristic dimension for this body is the side length, H .

In an experiment, a two-dimensional, square cross-section cylinder of $H=2\text{cm}$ is placed in a uniform on-coming flow of air at 10m/s . It is determined that the shedding frequency is $f=66\text{Hz}$. The experiment is repeated in water with a cylinder of size $H=5\text{cm}$. Assuming that the flows are similar (i.e. same Reynolds number), use the Buckingham Π -theorem to determine the shedding frequency in water. If, for the above conditions, drag per unit length of 3.74N/m is measured in water, what would be the drag per unit length in air for a speed of 10m/s and $H=2\text{cm}$? For air use $\rho=1.2\text{kg/m}^3$ and $\mu=1.8\times 10^{-5}\text{ kg/m-s}$; for water use $\rho=998\text{ kg/m}^3$ and $\mu=1.0\times 10^{-3}\text{ kg/m-s}$.



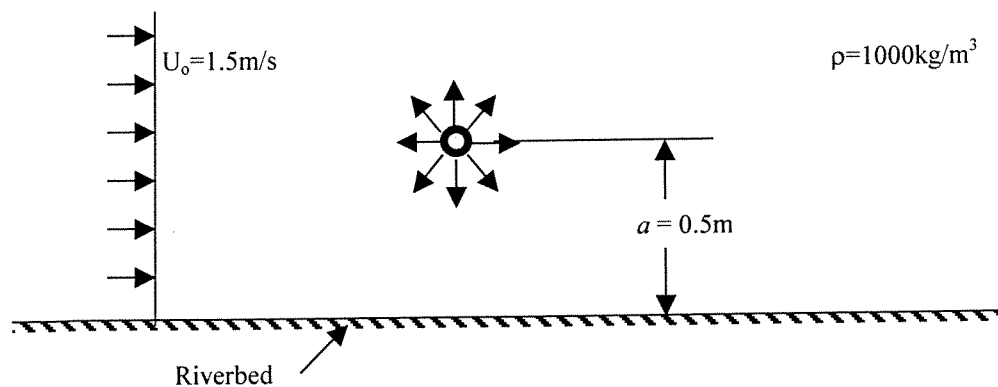
Question A3: A medical research centre is developing a new heart valve. In order to better understand the hydrodynamic loads on the valve generated by the blood flow ($\rho=1100 \text{ kg/m}^3$; $\mu=1.2 \times 10^{-3} \text{ kg/m-s}$), a consultant engineer is asked to investigate the process. The engineer determines that in order to obtain a good series of observations and measurements, a scale-up model of 5:1 is necessary. The model working fluid is chosen to be Helium ($\rho=0.166 \text{ kg/m}^3$; $\mu=1.94 \times 10^{-5} \text{ kg/m-s}$). If the blood flow rate in a typical heart is 0.1 litres per second and if the valve operates 80 times per minute, what must be the Helium flow rate and the operating frequency of the model valve? If the valve fails after 3.5 years in the laboratory set-up, how long would it be expected to last in actual use inside a human, assuming that the failure is due to hydrodynamic forces?

Question A4: Air ($\gamma=1.4$; $R=287 \text{ J/kg-K}$) flows isentropically through a converging nozzle of exit area 0.5 cm^2 . The nozzle is fed from a large plenum chamber, where the stagnation pressure and temperature are 500 kPa (abs) and 27°C , respectively. The exit pressure is 100 kPa (abs) . Determine:

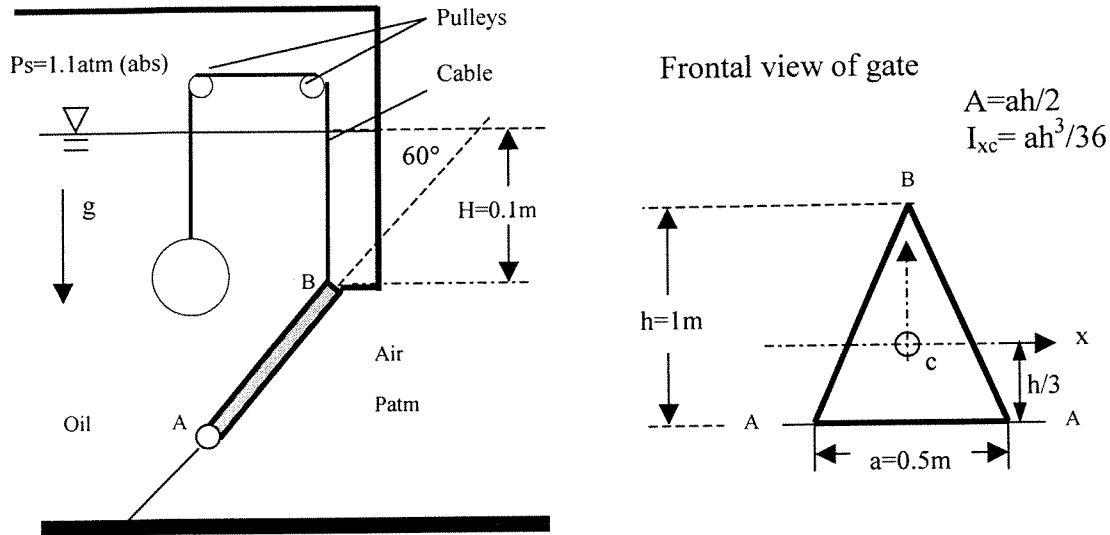
- The Mach number and pressure at the throat
- The Temperature at the throat
- The mass flow rate through the nozzle.

PART B: Answer any 3 of the following 5 questions.

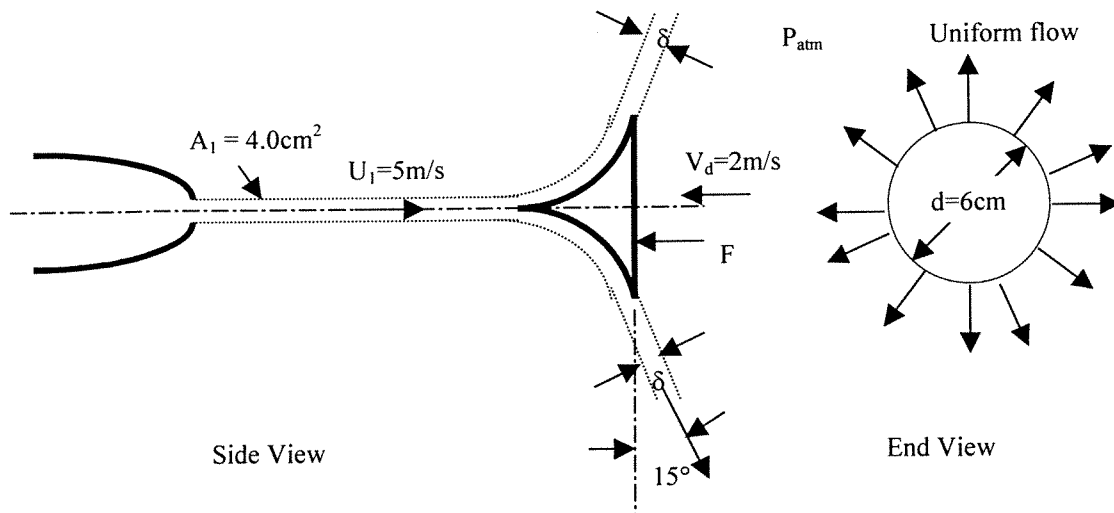
Question B1: A pipeline is submerged in a deep river. The centre of the pipe is located at $a=0.5 \text{ m}$ from the riverbed. Due to corrosion problems, the walls of the pipe become porous, resulting in a leak. The leak may be modelled as being uniformly distributed along the length of the pipe and the fluid is escaping at a rate of $q=1.0 \text{ m}^3/\text{s}$ per unit length of pipe. Assuming that the fluid is of the same density as water ($\rho=1000 \text{ kg/m}^3$), determine the net force per unit length on the pipe if the lake current results in a uniform speed of $U_0=1.5 \text{ m/s}$. Neglect gravitational and viscous effects. The riverbed can be assumed to be flat. The pipe diameter is very small compared to the separation a .



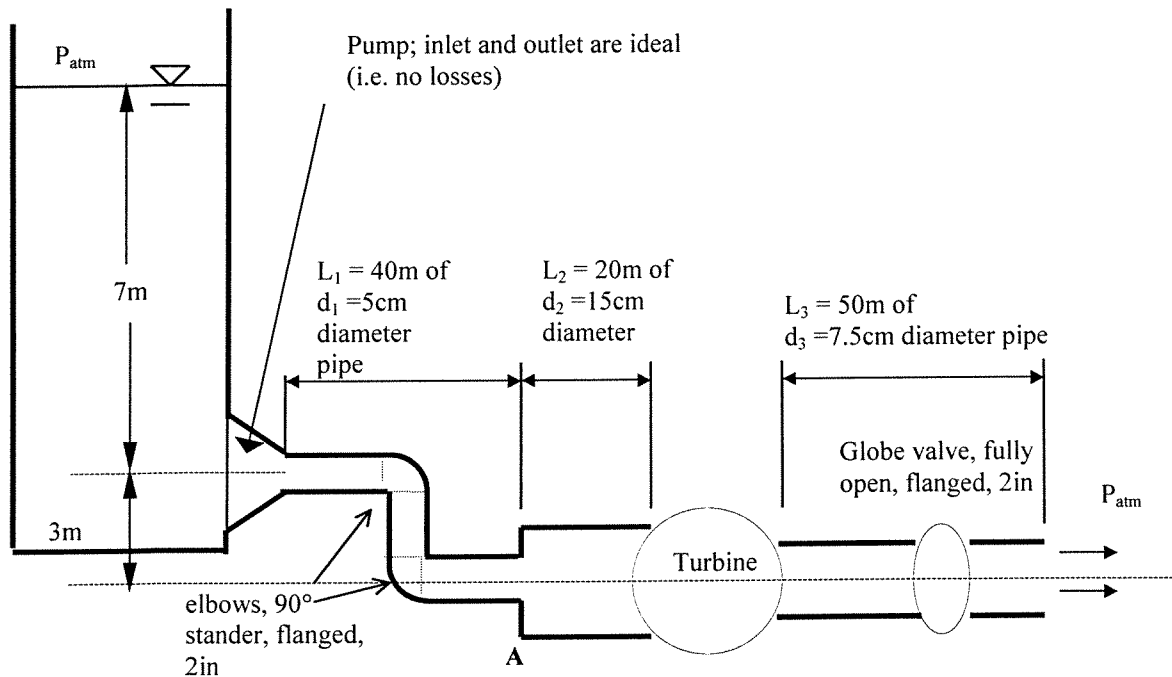
Question B2: A closed reservoir is filled with oil of density 800kg/m^3 . A triangular gate is located along a side face as shown in the figure below. The gate is inclined at 60° to the horizontal and is hinged along the line A-A. A solid metal sphere ($\rho=5500\text{kg/m}^3$) is suspended from a cable through frictionless pulleys and applies a force on the gate at B as shown. The pulley cable at B is vertical. What must be the minimum diameter of the sphere to keep the gate closed?



Question B3: Water impinges on a conical disc as shown in the diagram below. The water ($\rho=1000\text{kg/m}^3$) jet leaves the nozzle with a speed of $U_1=5 \text{ m/s}$ through an area of $A_1=4.0\text{cm}^2$. The disc is moving towards the nozzle with a speed of $V_d=2 \text{ m/s}$. The jet hits the disc and deflects uniformly and circumferentially. Given that the disc diameter, d , is 6 cm , calculate the jet thickness, δ , and the force, F , which must be applied on the disc. Neglect frictional forces on the plate and gravitational effects.



Question B4: Calculate the equivalent hydrostatic head produced by the pump for the system shown below if the flow rate is $0.004\text{m}^3/\text{s}$ and the turbine extracts energy from the flow at a rate of 910W . The system consists of 40m of 5cm -diameter pipe, 20m of 15cm -diameter pipe and 50m of 7.5cm -diameter pipe, all made of cast iron (equivalent roughness is 0.26mm). The two elbows are 90° standard, flanged bends (2in . nominal i.d.). The globe valve is fully open (flanged, 2in nominal i.d.). The sudden expansion, A, has a loss coefficient of 0.25 based on the inlet flow speed. What is the power necessary to operate the pump if the mechanical efficiency is 0.785 . For water, use the density $\rho=1000\text{ kg/m}^3$ and the kinematic viscosity $\nu=1.0 \times 10^{-6}\text{ m}^2/\text{s}$. The flow exhausts to atmosphere. The reservoir can be assumed to be very large.



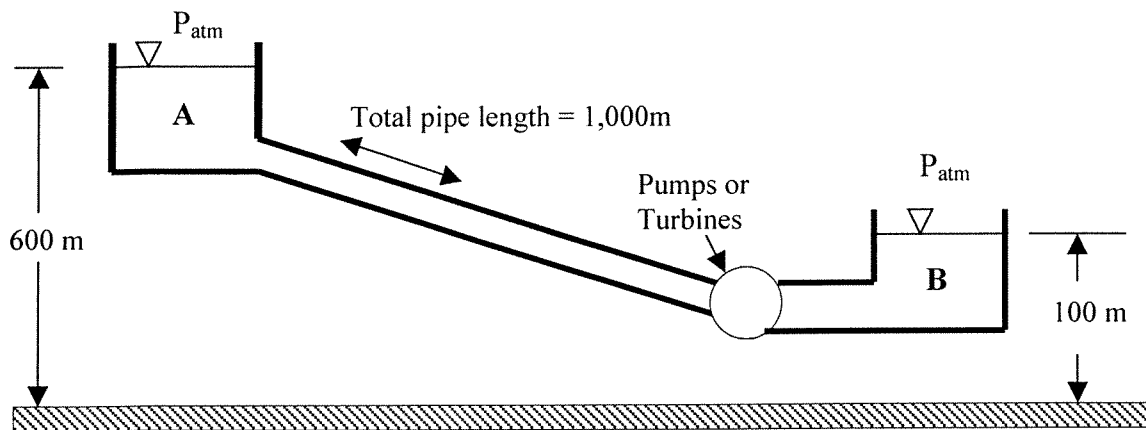
Question B5: A pumped-storage hydro-electric plant generates high value electricity during peak demand times, by allowing water (density $\rho=1000 \text{ kg/m}^3$; kinematic viscosity $\nu=1.0 \times 10^{-6} \text{ m}^2/\text{s}$) to run through a pipe from an elevated reservoir. It then uses low-cost, off-peak electricity to pump water back up into the upper reservoir (See diagram below).

The water surface in the very large reservoirs (A and B) is 600 m and 100 m, respectively, above datum. When generating power, water discharges from A to B through the turbine system at a total rate of $200 \text{ m}^3/\text{s}$ through a 3 m diameter concrete pipe (roughness value of $\epsilon = 0.4\text{mm}$). The pipe is 1,000 m in length. Later, the water is pumped back up to the upper reservoir at a rate of $100 \text{ m}^3/\text{s}$ through a 1,000 m long, 2 m diameter concrete pipe ($\epsilon = 0.4\text{mm}$).

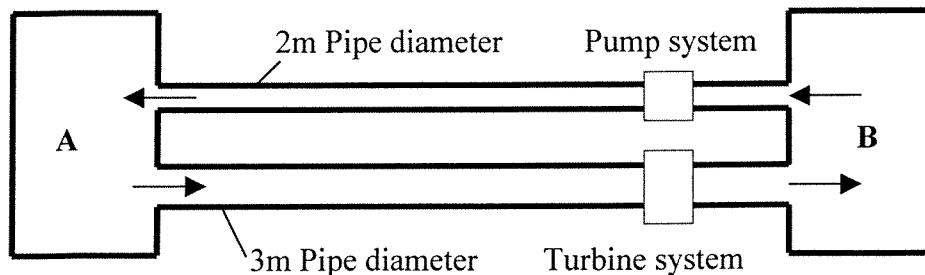
You are given that the efficiency of the turbine generators in converting fluid power to electricity is 80%, while the efficiency of the pump system in converting electricity into fluid power is 78%.

Considering only major losses, determine the electrical power output from the turbine generator and the required electrical power for the pump system.

ELEVATION VIEW:



PLAN VIEW:



ADDITIONAL INFORMATION

POTENTIAL FLOW:

	Potential Function	Streamline Function	Complex Potential
Uniform Flow:	$\phi = Ux$	$\psi = Uy$	$\Omega(z) = Uz$
Source:	$\phi = \frac{q}{4\pi} \ln(x^2 + y^2)$	$\psi = \frac{q}{2\pi} \tan^{-1}\left(\frac{y}{x}\right)$	$\Omega(z) = \frac{q}{2\pi} \ln z$
Vortex:	$\phi = \frac{\Gamma}{2\pi} \tan^{-1}\left(\frac{y}{x}\right)$	$\psi = -\frac{\Gamma}{4\pi} \ln(x^2 + y^2)$	$\Omega(z) = \frac{\Gamma}{2\pi i} \ln z$

Note: $z = x + iy$; $i = \sqrt{-1}$

HELPFUL INTEGRALS:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \quad ; \quad \int \frac{x dx}{x^2 + a^2} = \frac{1}{2} \ln(x^2 + a^2)$$

$$\int \frac{x^2}{(x^2 + a^2)^2} dx = \frac{1}{2a} \left[\tan^{-1}\left(\frac{x}{a}\right) - \frac{ax}{x^2 + a^2} \right]$$

MAJOR LOSSES

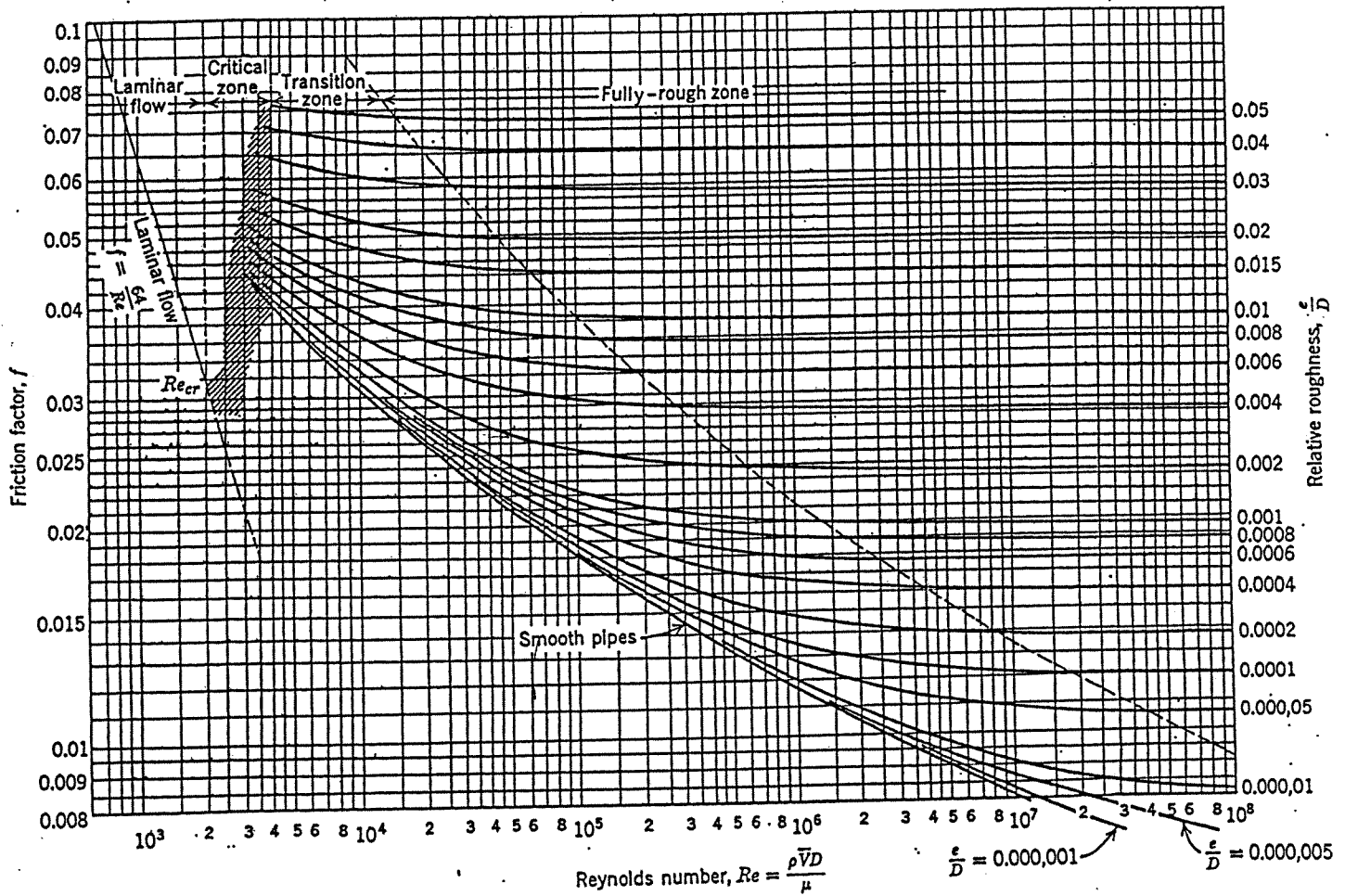
FRICITION COEFFICIENTS:

Colebrook Equation:
$$\frac{1}{f} = -2.0 \log_{10} \left[\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right]$$

Laminar Flow Equation:
$$f = \frac{64}{Re} \quad Re < 2000$$

$$Re = \frac{\rho U D}{\mu} = \frac{U D}{\nu}$$

MOODY CHART for PIPE FRICTION



MINOR LOSSES

Typical loss coefficients for commercial pipe fittings

Nominal Diameter (in.)	Screwed or Soldered				Flanged				
	$\frac{1}{2}$	1	2	4	1	2	4	8	20
Valves (fully open)									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows									
45° standard	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° standard	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° standard	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

AVERAGE ROUGHNESS OF COMMERCIAL PIPES

Material (new)	ϵ	
	ft	mm
Riveted steel	0.003-0.03	0.9-9.0
Concrete	0.001-0.01	0.3-3.0
Wood stave	0.0006-0.003	0.18-0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015
Glass	"Smooth"	"Smooth"