National Exams December 2003

98 – Chem – B3: Simulation, Modelling and Optimization

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any questions, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a CLOSED – BOOK exam, but candidates may use (a) one of two calculators: the CASIO approved models, or the SHARP approved models; (b) a scaled (cm or inch) ruler.

3. Any five of the seven questions provided constitute a complete paper. If more than five questions are answered, the exam grade will be made up of the five highest marks.

4. All questions are of equal value.

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Q1. Cylindrical soup cans are to be made out of Zn – Al alloy sheets at a cost of c \$/unit area. The volume of the cans is a constant. Find the dimensions (i.e. diameter and length) of the cans which minimize the cost of production, in terms of the constant volume. Note that the bottom and the top of a can are sealed by the same material.

Q2. An optimization problem yields the function $f(x, y) = x^3 - 12xy + 8y^3$, without constraint. (a) How many critical points does this function possess? (b) What is the nature of the critical points?

Q3. The profit of a certain operation may be expressed as $P = 2X_1 + 3X_2 - X_3$ with appropriate constraints; X_1, X_2 and X_3 are operating variables. The final simplex tableau is as follows. Note that X_4, X_5, and X_6 are slack variables, and $X_i \geq 0, i = 1,\ldots,6$

X_1	X_2	X_3	X_4	X_5	X_6	P	
0	1	5/7	3/7	-1/7	0	0	9/7
1	0	4/7	1/7	2/7	0	0	24/7
0	0	12/7	- 11/7	-1/7	1	0	65/7
0	0	30/7	11/7	1/7	0	1	75/7

(a) What is the algebraic form of the profit equation at this stage?
(b) What is the optimal profit?

Q4. Merit ratings of 41 employees of a chemical company after 3 years of service (Y) are to be correlated against the score an employee achieved during the entrance test (x_1), and the verbal score the same employee obtained at a test after 1.5 years of service (x_2). The proposed regression has the form

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$$

with the following numerical information available: (a) sum of the squares due to regression = 712.0; (b) sum of the squares due to residual (or error) = 1666.0

(a) Establish the (standard) ANOVA table and test the null hypothesis that Y does *not* depend on x_1 and x_2, at a significant and a highly significant level of confidence.
(b) Establish the narrowest range of the P – value of the test, using the critical score tabulation below.

Confidence level	0.25	0.10	0.05	0.025	0.01	0.005
Critical F	1.386	1.997	2.450	2.904	3.514	3.986

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(c) The regression has $R^2 = 0.729$. Explain if this is, or is not, a statistically good regression.

(d) Assume that the C – matrix [$\mathbf{C} = (\mathbf{X}^T\mathbf{X})^{-1}$; **X** is the independent variable-based array] , and the error variance are known. Describe how you would test the null hypothesis that there is no interaction between x_1 and x_2.

Q5. A pollutant diffuses vertically into a long an narrow aquifer channel with an apparent diffusion coefficient D_a= 0.01 cm²/s. If the unsteady – state diffusion model is adopted, the concentration distribution of the pollutant in the aquifer is given by

$$\frac{c}{c_s} = 1 - (2/\pi)\int_0^z \exp(-u^2)du; z \equiv \frac{x}{2(D_a t)^{1/2}}$$

where x is the vertical distance measured from the aquifer surface, t is time and c_s is the pollutant concentration on the surface, assumed to be a constant.

What is the *approximate* fractional pollutant concentration at a 15 cm depth, after 200 hours of diffusion, using an appropriate mathematical technique of your choice?

Q6. In a biochemical process, the growth of bacteria has been described by a discrete – variable model

$$y_{k+2}y_k^2 = y_{k+1}^3; y_1 = 1; y_2 = 5$$

k, k+1, k+2, … represent coded time instants, and y_k is the coded bacteria concentration at the k-th time interval.

(i) Linearize the model and obtain a closed form solution for y_k [i.e. $y_k = f(k)$].
(ii) What is the predicted coded concentration at the fourth coded time instant?

Q7. A thin nickel plate initially at a temperature of 323 kelvin is suddenly inserted into the centre of an oven whose inside walls are kept at 800 kelvin. The increase in plate temperature with time is given by the thermal balance

$$\frac{dT}{dt} + 2.8189x10^{-11}(T^4 - 800^4) = 0 \quad [T] = K; [t] = s.$$

Obtain a linear balance approximating the T(t) relationship for short times. Given the rigorous values of T shown below for selected time instants, comment on the goodness of the linear approximation. Selected rigorous values are: 324.2 K at 0.1 s, 334.2 K at 1.0 s, 378.6 K at 5.0 s, and 432.4 K at 10 s. [obtainable analytically or by computer].