

PROFESSIONAL ENGINEERS OF ONTARIO

National exams / December 2003

98-CIV-B9

Applications of the Finite Element Methods

Time: 3 hrs

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Answer all questions.

20 questions @ 5 marks = 100 marks

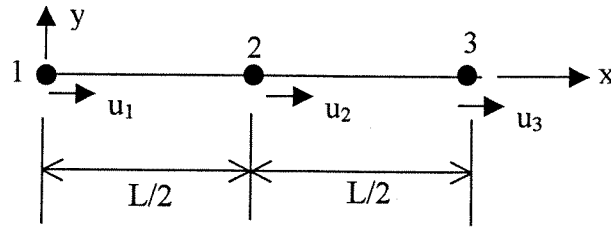
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NOTES:

If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumption made.

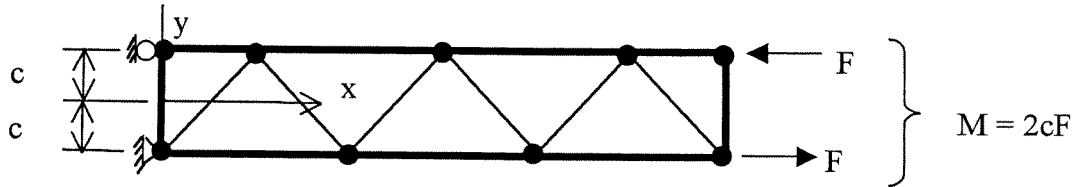
This is a **closed book** exam. Candidates may use one of two calculators, the casio or sharp approved models.

- The three-node bar element shown is uniform and is allowed only axial displacement  $u$ . Its nodal degrees of freedom are  $u_1, u_2$  and  $u_3$ . Derive its shape functions.

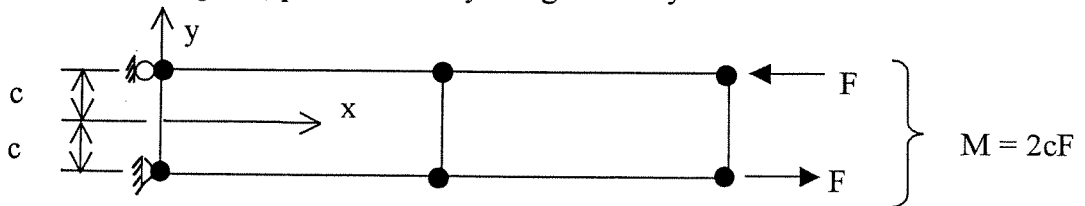


- A pin-jointed plane truss is modeled by plane frame elements. If rotational degrees-of-freedom are suppressed at all nodes, is the truss correctly modeled? Explain.

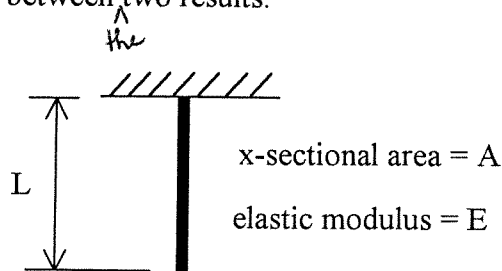
- The cantilever beam shown is modeled by an extremely coarse mesh of constant-strain triangles. The beam is loaded by an end moment,  $M$ . Qualitatively plot the variation of  $\sigma_x$  versus  $y$  along the  $x$ -axis.



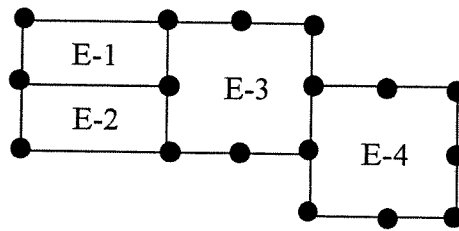
- A cantilever beam is modeled by bilinear plane elements, as shown. For the loading condition given, plot  $\sigma_x$  and  $\tau_{xy}$  along the line  $y=0$ .



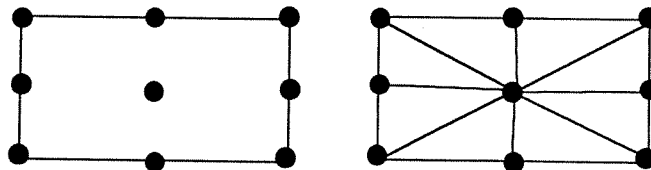
- The uniform bar shown hangs under its own weight  $W$ . Compute the deflection of the lower end in terms of  $W, L, A$ , and  $E$  using (i) one element of length  $L$  and (ii) two elements (each of length  $L/2$ ). The upper most node is fixed in both cases. Explain the difference between two results.



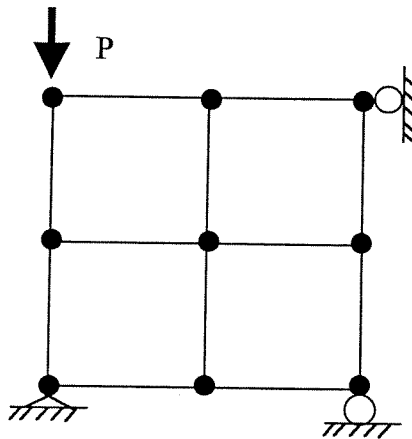
6. Identify the defects associated with connecting four-node and eight-node elements in the pattern shown in the following.



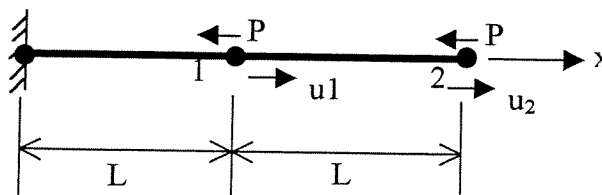
7. As an alternative to the nine-node element in the following, the physical domain can be mapped with eight linear triangles. What are the comparative advantages and disadvantages of these two alternatives ?



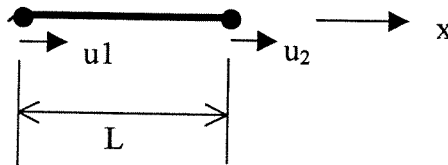
8. The plane structure shown is built with four bilinear elements, each integrated by one-point Gauss quadrature. Sketch the possible unstable (zero energy) collapse modes of the structure.



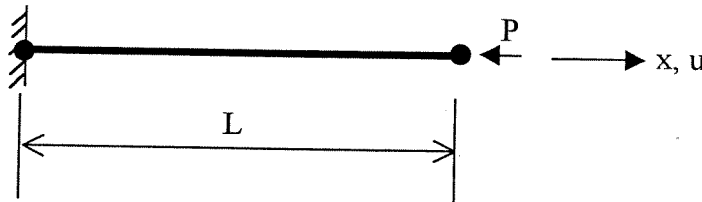
9. Discuss the merits and demerits/limitations of using substructuring technique in the analysis of large structural systems.
10. A structure comprises of two identical bar elements, each of axial stiffness  $k=AE/L$  as shown in the following. Assemble global equilibrium equations of the discretized structure with a penalty constraint to impose  $u_1=u_2$ .



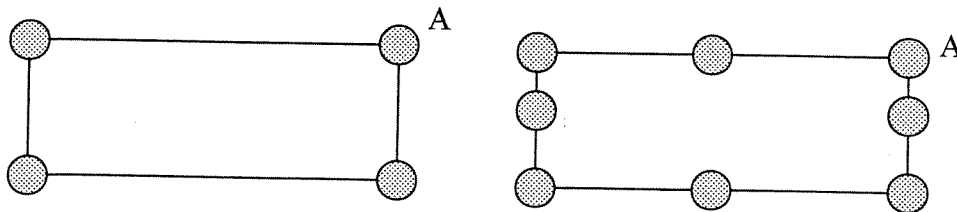
11. State the differences in formulations of Mindlin plate and Kirchoff plate elements. Explain which element would you use under what circumstances, and why?
12. Each side of a rectangular box is modeled with flat shell elements and a uniform hydraulic pressure is applied on the shell surfaces. Along the edges where the sides intersect, what degrees-of-freedom can probably be set to zero, and why?
13. A uniform two-node bar element has cross-section area  $A$ , length  $L$ , and mass density  $\rho$ . Calculate the consistent mass matrix of the element. Can a diagonal coefficient in a consistent mass matrix ever be negative in any finite element? Explain.



14. A homogeneous elastic beam element is fixed at one end and is loaded statically by  $P$  as shown in the following figure. If the load  $P$  is increased indefinitely, do you expect the beam element to buckle at any point while solving the static equilibrium equation under incremental load applications? Justify your answer.



15. Draw the appropriate shape functions for the corner node A in following elements. How will the strain vary over these element domains? Explain.



16. Discuss how you would use the finite element stress-strain analysis results to assess the safety against impending failure of ductile (e.g. mild steel) and brittle (e.g. concrete) materials.

17. Discuss the implications of shear strain on the behavior of following four-node planes-stress element.

Given, displacement at any point inside the element:

$$u = \sum H_i u_i$$

where, interpolation functions,  $H_i$ , are given by:

$$H_1 = (x-a)(y-b)/4ab$$

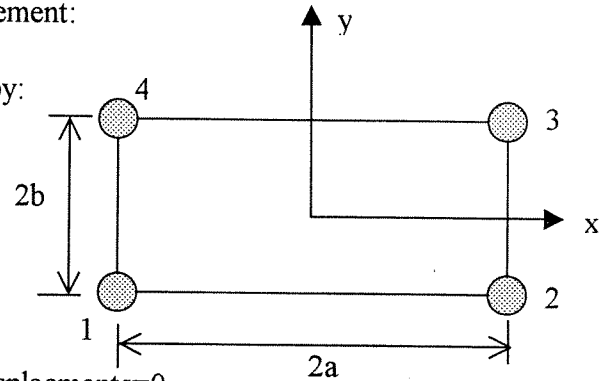
$$H_2 = -(x+a)(y-b)/4ab$$

$$H_3 = (x+a)(y+b)/4ab$$

$$H_4 = -(x-a)(y+b)/4ab$$

and the nodal displacements,  $u_i$ , are:

$$u_1^x = -k; u_2^x = +k; u_3^x = -k; u_4^x = +k; \text{ all other displacements} = 0.$$

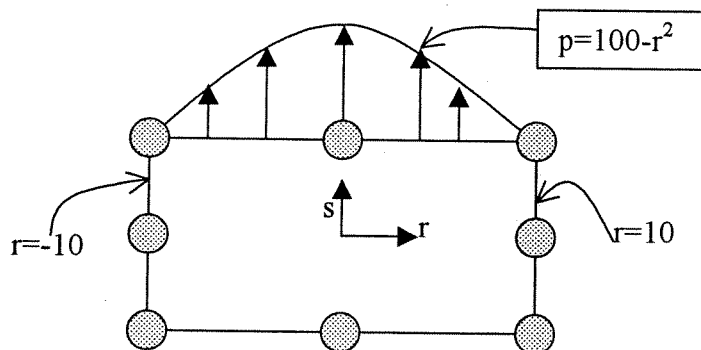


18. Distortion energy in a 3-D solid (per unit volume) is given by:

$$U_d = (1+\nu)[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]/6E$$

where  $\sigma_1, \sigma_2$  and  $\sigma_3$  are principal stresses,  $\nu$  is Poisson's ratio and  $E$  is the elastic modulus. Assuming that at impending failure state, the distortion energy per unit volume of a 3-D solid element is same as that in the yielding failure of a uniaxial tensile test specimen, find out the definition of von Mises stress.

19. Calculate the consistent nodal loads on the following element?



20. Explain one possible technique for the error estimation of finite element stress analysis results.