National Exams - December 2003
98-Elec-B11, Electro-Optical Engineering

3 hours duration

NOTES:

1. If doubt exists as to the proper interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement about any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.
3. This is a "**Closed-Book**" examination. The candidate may have a single 8.5 inch by 11 inch sheet (both sides) of hand-written notes as an aid for the examination.
4. Any **five** questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
5. All questions are of equal value.
6. This examination paper has 4 pages.

Values of common constants:

$\varepsilon_o = 8.854 \times 10^{-12}$ F/m
$\mu_o = 4\pi \times 10^{-7}$ H/m
$c = 2.998 \times 10^{8}$ m/s
$q = 1.602 \times 10^{-19}$ C
$h = 6.626 \times 10^{-34}$ J•s
$K = 1.381 \times 10^{-23}$ J/ °K
0°K = -273°C
1 Å = 1.0 x10^{-10} m

Si $\varepsilon_r = 11.8$
Si $n = 3.42$
Si $E_g = 1.11$ eV
Ge $\varepsilon_r = 16.0$
Ge $n = 4.01$
Ge $E_g = 0.67$ eV
GaAs $\varepsilon_r = 13.2$
GaAs $n = 3.63$
GaAs $E_g = 1.41$ eV
InGaAsP $n = 3.5$

$LiNbO_3$ $\varepsilon_r = 32$
$LiNbO_3$ $n_o = 2.30$

Useful formulas:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

$$P(n) = \frac{N^n \exp(-N)}{n!}$$

$$I_s = R_o\sqrt{P_o P_1}\cos\theta$$

$$n(E) = n_0 - \frac{1}{2} r n_0^{\,3} E$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 1

A multimode fiber has core diameter 100 μm, core refractive index 1.475, and a cladding refractive index of 1.455 . The fiber is operated at λ=850 nm.

(a) Calculate the V-number for the fiber and estimate the number of modes.
(b) Calculate the wavelength beyond which the fiber becomes single mode.
(c) Calculate the numerical aperture of the fiber and the maximum acceptance angle.
(d) For NRZ signaling, calculate the modal dispersion $\Delta\tau$ and hence the BL product given that the rms dispersion is $\sigma=0.29\ \Delta\tau$ in which $\Delta\tau$ is measured at FWHM.
(e) Explain why modal dispersion is reduced in a GRIN fiber compared with the step-index fiber.

Question 2

A Ge pn-junction photodiode has a measured responsivity shown in Figure 1. Its photosensitive area is 0.008mm^2. It is used under reverse bias of 10V when the dark current is 0.3μA and the junction capacitance is 4 pF. The transit time of the photodiode is 0.5 ns.

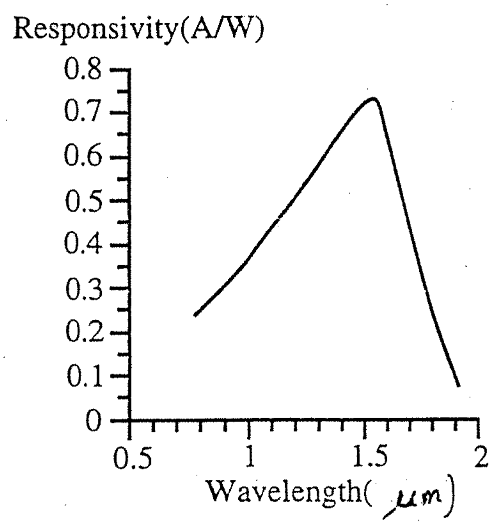

Figure 1. Responsivity of Ge photodiode.

(a) Calculate the quantum efficiency at 850, 1300, and 1550 nm.
(b) What is the intensity of light at 1550 nm that gives a photocurrent equal to the dark current?
(c) What would be the effect of lowering the temperature on the responsivity curve?
(d) Given that the dark current is in the range of microamperes, what would be the advantage in lowering the temperature?
(e) Suppose that the photodiode is used with a 100Ω resistance to sample the photocurrent. What is the bandwidth for detection?
(f) Explain the difference between the photovoltaic mode and the photoconductor mode of operation of a detector diode. What are the advantages and disadvantages of each?

Question 3

(a) Explain why population inversion is essential to the operation of a laser diode.

(b) Show that the ratio between the output power P_1 from mirror 1 with a reflectance R_1 and the output power P_2 from mirror 2 with reflectance R_2 in a Fabry-Perot cavity of a laser operating in steady-state is given by

$$\frac{P_2}{P_1} = \frac{1-R_2}{1-R_1}\sqrt{\frac{R_1}{R_2}}$$

(c) Consider a semiconductor Fabry-Perot optical cavity of length 200 μm with end mirrors that have reflectance 80%. The refractive index of the semiconductor is 3.7 Calculate the cavity mode nearest to the free space wavelength of 1300nm.

(d) For the same cavity, calculate the frequency separation of the modes, the finesse of the cavity, and the spectral width of each mode in GHz and in nm.

Question 4

A PCM-ASK system having a data transmission rate 250 Mb/s at a wavelength 1.3 μm must have a signal-to noise ratio of 13 dB in the detected signal for a BER=10^{-9}. The quantum efficiency of the PIN photodiode is 0.65 . Neglect amplifier noise. The operating temperature is 27°C. Assume the signal threshold is set half-way between the two signal levels for a detected "1" and "0". Calculate the sensitivity of the system for the following three different detection schemes.

(a)An ideal shot-noise limited direct detector.

(b) Direct detection, including thermal noise from a load resistor of 1K Ω.

(c) A homodyne detector, with local oscillator power of 0 dBm and phase error of 20 degrees.

Question 5

(a) Explain how photoconductive gain occurs in a photoconductor.

(b) Consider a GaAs photoconductor in which carriers are generated at a rate of $G_L = 10^{20}$ $cm^{-3}\,s^{-1}$. The device area is 10 μm x 10 μm and the length is 10 μm. The material parameters of the device are:

Background doping, $N_d = 10^{16}$ cm^{-3}; $N_a = 0$
Applied voltage, $V_{app} = 1.0$ Volt
Electron velocity at the applied field, $v_e = 6 \times 10^6$ cm/s
Hole velocity at the applied field, $v_h = 10^6$ cm/s
Electron lifetime, $t_e = 10^{-7}$ s
Hole lifetime, $t_h = 10^{-8}$ s

Calculate,

(i) the excess carrier concentration
(ii) the steady state photocurrent
(iii) the photoconductive gain.

(c) Explain why the gain bandwidth product is almost constant for a photoconductor.

Question 6

A digital fiber optical communication system using NRZ (non return-to-zero) light pulses at a wavelength of 1.55 μm, is going to be used to send signals with a bit rate between 100 Mbits/s and 10 Gbits/s at a maximum BER (bit-error-rate) value of 10^{-12}. The maximum allowed pulse broadening is half a bit period and the system security margin should be 5 dB. The following components are available to build the system.

Laser:
InGaAsP DFB laser with an output power of 10 mW and spectral width of 1 Å.
Coupling loss from laser to fiber is 5 dB.

Fiber:
Single-mode silica glass fiber with dispersion 25ps/(km-nm) and loss 0.2dB/km

Detector:
InGaAsP - pin diode,
Coupling loss from fiber to detector is 3 dB.
Needs a minimum of 500 photons/bit to achieve a BER value of 10^{-12}.

a) Make a log (y=fiber length) - log(x = bit rate) sketch of the maximum allowed fiber length due to the attenuation limit and due to the dispersion limit for bit rates at 10Mb/s, 100 Mb/s, 1Gb/s and 10 Gb/s.
b) For what fiber length (and at what bit rate), is the system limit due to attenuation and dispersion the same?
c) What is the theoretical limit for the minimum number of photons/bit that is needed to get a BER value of 10^{-12} ?

Question 7

A longitudinal electro-optic Pockel cell modulator of *LiNbO₃* is designed for modulation of the red laser line (635 nm) from a HeNe laser. The crystal has cylindrical geometry with a diameter of 20 mm and a length of 30 mm. The *RC* limited bandwidth of the modulator is 100 MHz, where *R* is the load resistance and *C* is the (parallel plate) capacitance of the crystal.

(a) Make a sketch of the Pockel-cell modulator together with the necessary external components that are needed. Briefly explain how the modulation is achieved.
(b) Calculate the minimum "half-wave" voltage needed for the modulator.
(c) What power does the sinusoidal signal generator need to operate the modulator at 100% modulation index?
(d) What is the main advantage of a transverse electro-optic Pockel-cell modulator?