NATIONAL EXAMS December 2003
98-Elec-B3 Advanced Control Systems

3 hours duration

NOTES:

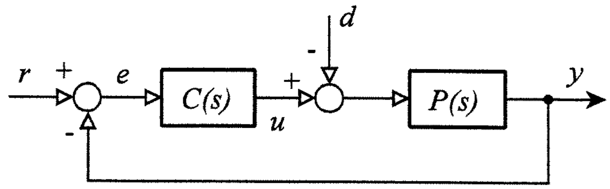
1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio FX-991 or a Sharpe EL-540. This is a closed-book examination. Tables of Laplace and z-transforms will be supplied.

3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1. Consider the feedback system with $P(s)$ and $C(s)$ as defined.



$$C(s) = \frac{sK_p + K_i}{s}$$

$$P(s) = \frac{1}{(s+2)(s+3)}$$

(a) Find the range of K_p and K_i for closed loop stability.

(b) With $K_i = 0$, find the value of $K_p = K_{po}$ such that the steady state value of $e(t) = 0.05$ when $d(t) = 0$ and $r(t)$ is a unit step.

(c) With $K_p = 0$, the value of K_i is increased to a value of K_{imax} at which the system exhibits sustained oscillation. What is the frequency of oscillation?

(d) For $K_i = K_{imax}$ and $K_p = K_{po}$ find an expression for the steady state value of u(t) as $t \to \infty$ when $d(t) = 2$, and $r(t)$ is a ramp with unit slope.

2. Consider the system,

$$G(s) = \frac{2(1+s)}{s(1+0.2s)(1+0.5s)}$$

(a) Find a state space model for the system.

(b) The system input and output are uniformly sampled with a sample period of h and the discrete input is applied to a zero order hold device. Identify the poles of the sampled data system as a function of h.

(c) Find the state transition matrix of the sampled data system.

3. Consider the open loop system,

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

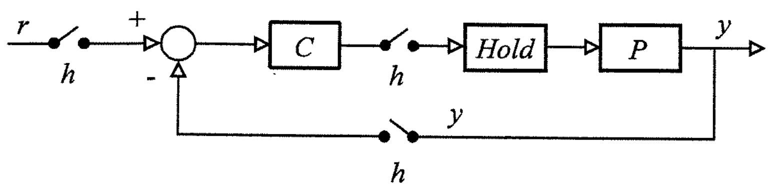
$$C = [\,1 \quad 1 \quad 0\,], \quad D = 0$$

(a) Justify whether the system is controllable and observable.

(b) Justify whether the system is stable or unstable in the bounded input bounded output sense.

(c) Assume all of the states, $x(t)$, are available for measurement. Find a statefeedback controller such that the closed loop poles are located at $s = -2, -3 \pm j$.

4. Consider the sampled data system with uniform sample period, $h = 1$ second.



$$C(z) = \frac{K}{z - 1}$$

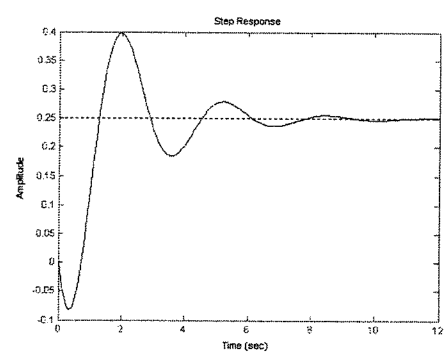
$$P(s) = \frac{1}{10s + 2}$$

(a) Find the discrete time transfer function that relates $Y(z)$ to $R(z)$.
(b) If $r(t) = 2 \sin t$, find an expression for $y(kh)$ in steady state when $K = 1$.
(c) Justify whether or not the closed loop system is stable when $K = 4$.

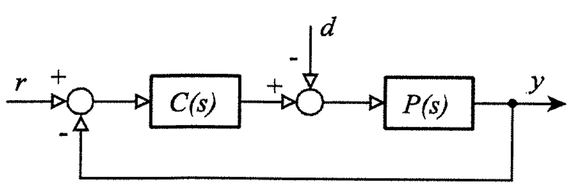
5. Several experiments are conducted on an unknown system:

When a step of magnitude 2 is applied to the input, the steady state output is 10.
When a sinusoid of frequency 8 rad/sec is applied, the phase lag at the output is 90°.
When a sinusoid of frequency 4 rad/sec is applied, the phase lag at the output is 30°.

(a) Assume the system is second order system and has no finite zeros. Find the parameters of the second order model.
(b) For the model identified in (a) determine the maximum overshoot for a unit step input.
(c) Another system has the step response shown below.
What can you say about the poles and zeros of the system?



6. Consider the feedback system,



$$C(s) = \frac{K(s\tau + 1)}{s}, \quad P(s) = \frac{e^{-sT}}{s}$$

(a) For $T = 0$ and $K = 1$ find the value of τ such that the phase margin is 60°.
(b) For $K = 1$ and the value of τ obtained in (a) above, sketch the shape of the polar plot, i.e. the Nyquist diagram, for the loop transfer function when $T = 0$. Then show how the shape changes when T is a value greater than zero.
(c) For the value of τ above and $T = 0.1$ seconds find the value of K for a gain margin of 2.

Inverse Laplace Transforms	
F(s)	***f(t)***
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta} + \dfrac{C-jD}{s+\alpha-j\beta}$	$e^{-\alpha t}(2C\cos\beta t + 2D\sin\beta t)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^n e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{(s+\alpha+j\beta)^{n+1}} + \dfrac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\dfrac{t^n e^{-\alpha t}}{n!}(2C\cos\beta t + 2D\sin\beta t)$

Inverse z-Transforms	
F(z)	***f(nT)***
$\dfrac{Kz}{z-a}$	Ka^n
$\dfrac{(C+jD)z}{z-re^{j\varphi}} + \dfrac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^n(C\cos n\varphi - D\sin n\varphi)$
$\dfrac{Kz}{(z-a)^r}$ $\quad r = 2, 3 \ldots$	$\dfrac{Kn(n-1)\ldots(n-r+2)}{(r-1)!\,a^{r-1}}a^n$

Table of Laplace and z-Transforms		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{-\alpha T}}$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\cos\beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta T)}{z^2-2z\cos\beta T+1}$
$\sin\beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta T}{z^2-2z\cos\beta T+1}$
$e^{-\alpha t}\cos\beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha T}\cos\beta T)}{z^2-2ze^{-\alpha T}\cos\beta T+e^{-2\alpha T}}$
$e^{-\alpha t}\sin\beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha T}\sin\beta T}{z^2-2ze^{-\alpha T}\cos\beta T+e^{-2\alpha T}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zT\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha T})$