

National Exams December 2004  
98-BS-1, Mathematics  
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
  2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
  3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
  4. All questions are of equal value.
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Marking Scheme:

1. 20 marks
2. (a) 4 marks, (b) 16 marks
3. 20 marks
4. 20 marks
5. (a) 5 marks, (b) 5 marks, (c) 10 marks
6. (a) 4 marks, (b) 4 marks, (c) 12 marks
7. (a) 3 marks, (b) 3 marks, (c) 14 marks
8. (a) 12 marks, (b) 8 marks

1. Find the volume of the region bounded by the paraboloid  $z = \frac{7}{4} + \frac{1}{4}(x^2 + y^2)$  and the plane  $z = 4$  that lies outside the cone  $z^2 - 4x^2 - 4y^2 = 0$ .
2. Consider the quadratic form  $5x^2 + 24xy - 5y^2 = 13$ .
  - (a) What type of conic section is represented by the above quadratic form?
  - (b) Transform the quadratic form to principal axes.
3. Find the general solution to the differential equation

$$y'' + 9y = 36t \sin(3t)$$

Note that ' denotes differentiation with respect to  $t$ .

4. Find the centre of mass of the solid bounded by the two paraboloids  $z = 2x^2 + 2y^2$  and  $z = 3 - x^2 - y^2$  whose density is  $\rho(x, y, z) = 2z$ .
5. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ -10 & -4 & -2 \end{pmatrix}$$

- (a) Show that  $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$  is an eigenvector of  $A$  and find the associated eigenvalue.
  - (b) Show that 3 is an eigenvalue of  $A$  and find an associated eigenvector.
  - (c) Solve the linear system  $\mathbf{x}' = \mathbf{A}\mathbf{x}$  for the function  $\mathbf{x}(t)$ .
6. Let  $F(x, y, z) = xy^2 + 3yz + z^3$ ,  $\mathbf{u} = \frac{1}{\sqrt{6}}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}$ , and  $P = (-1, 2, 1)$ .
    - (a) Find the gradient of  $F$  at the point  $P$
    - (b) Find the derivative of  $F$  in the direction of  $\mathbf{u}$  at the point  $P$
    - (c) Find the equation of the plane tangent to the surface  $F(x, y, z) = 3$  at the point  $P$ .
  7. Let  $C$  be the curve formed by the intersection of the cylinder  $x^2 + y^2 = 9$  and the plane  $z = 1 + y - 2x$ , travelled clockwise as viewed from the positive  $z$ -axis, and let  $\mathbf{v}$  be the vector function  $\mathbf{v} = 4z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$ .
    - (a) Evaluate the divergence of  $\mathbf{v}$
    - (b) Evaluate the curl of  $\mathbf{v}$
    - (c) Evaluate the line integral  $\oint_C \mathbf{v} \cdot d\mathbf{r}$ .
  8. Let  $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ .
    - (a) Find the linear approximation to  $f$  at  $(2, 3, 6)$ .
    - (b) Use this to approximate  $\sqrt{(1.97)^2 + (3.02)^2 + (5.98)^2}$