National Exams May 2004
98-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 5 marks, (b) 5 marks, (c) 5 marks, (d) 5 marks
2. 20 marks
3. 20 marks
4. (a) 12 marks, (b) 4 marks, (c) 4 marks
5. 20 marks
6. (a) 10 marks, (b) 10 marks
7. 20 marks
8. (a) 3 marks, (b) 3 marks, (c) 14 marks

1. Let P be the plane passing through the three points (0,1,4), (1,1,3) and (0,2,2).
   (a) Show that these three points do not lie on the same line.
   (b) Give a parametric representation for the plane P.
   (c) Find a vector normal to the plane P.
   (d) Find the line of intersection between the plane P and the plane

$$y + z = 1$$

2. Solve the initial value problem

$$y'' + 4y = 6\cos(2t), \qquad y(0) = 1,\ y'(0) = 0.$$

3. Find the general solution, $y(x)$, of the differential equation

$$2x^2y'' - 5xy' + 3y = 2x^3.$$

   Note that $'$ denotes differentiation with respect to x.

4. Consider the two lines defined as follows:
   $x = 3 + 2t, \quad y = 3, \quad z = 1 - t,$ (parameter t);
   $x = s, \quad y = 1 - 2s, \quad z = 2 + s,$ (parameter s).
   (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.
   (b) Find a third line orthogonal to both lines.
   (c) Is there a plane containing both lines? If so, find an equation for that plane.

5. Find the volume of the solid region below the plane $z = 4$ and above the parabola $z = 2x^2 + 2y^2 - 4$.

6. (a) Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$

(b) Solve the system of differential equations

$$\frac{dx}{dt} = 4x + 2y,$$
$$\frac{dy}{dt} = 3x - y + e^{-2t}.$$

7. Find the absolute maximum and minimum values of

$$f(x, y, z) = 3x + 2y^2 + z$$

over the elipsoid $3x^2 + y^2 + z^2 = 1$.

8. Let S be the boundary of the region enclosed by the cone $z = 5 - \sqrt{x^2 + y^2}$ and the plane $z = 1$ and let

$$\mathbf{F}(x, y, z) = x^2 z\mathbf{i} - xz\mathbf{j} + 2xy\mathbf{k},$$

(a) Evaluate the divergence of $\mathbf{F}$

(b) Evaluate the curl of $\mathbf{F}$

(c) Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{n}$ is the unit outward normal on S.