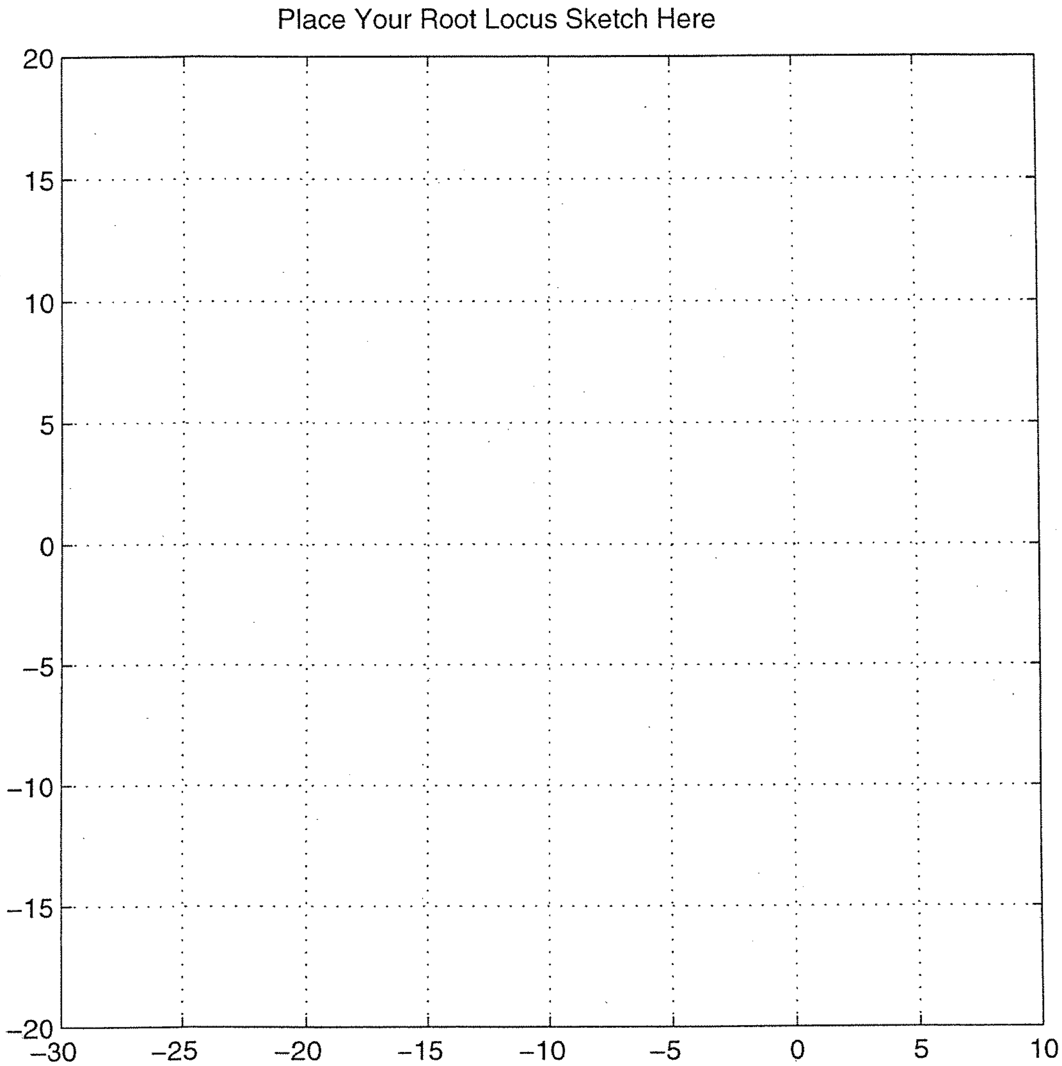
Place Your Root Locus Sketch Here
20
15
10
5
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10

Figure Q4.1

2) **(5 marks)** Find the value of the Proportional Gain K_p such that the closed loop system exhibits a damping ratio of $\zeta = 0.707$. For this value of the gain, briefly explain whether the closed loop dynamics can be adequately represented by a second order system model. If it can, find the appropriate parameters of the model: K_{dc}, ω_n, ζ. Complete Table Q4.2.

Table Q4.2

Damping ratio of the model is:	$\zeta =$
Frequency of natural oscillations of the model is:	$\omega_n =$
DC gain of the model is:	$K_{dc} =$
Model transfer function is:	$G_m(s) =$

3) **(5 marks)** For the same value of the gain, evaluate the following specifications of the closed loop step response: percent overshoot PO, settling time $T_{settle5\%}$ and steady state error $e_{ss\%}$. Complete Table Q4.3.

Table Q4.3

Percent Overshoot is:	P.O. =
Settling time is:	$T_{settle5\%} =$
Steady state error (in %) is:	$e_{ss\%} =$

Question 5

Consider a control system described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4 & -4 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

1) (**12 marks**) For the system in question do the following:
   - Find the system transfer function, $G(s) = \dfrac{Y(s)}{U(s)}$

HINT: Avoid complicated matrix manipulations in derivation of results and use canonical properties of the system instead.

- Find the system eigenvalues
- Check the system controllability and observability

HINT: Avoid complicated matrix manipulations in derivation of results and use canonical properties of the system instead.

2) (**4 marks**) Assume the state feedback of the form:

$$u = K \cdot \mathbf{x} + r \qquad \text{where: } \mathbf{K} = [k_1 \quad k_2 \quad k_3 \quad k_4]$$

and use pole placement by state feedback to place the compensated system eigenvalues at: -3, -4, -5 and -6. What are the gain values in the vector **K** ?

3) (**4 marks**) Write the closed loop transfer function of the system, $G_{cl}(s) = \dfrac{Y(s)}{R(s)}$

Question 6

A unit feedback control system is shown in Figure Q6.1. Frequency response plots of the open loop transfer function when Proportional Gain is set to $K_p = 1$ are shown in Figure Q6.2.

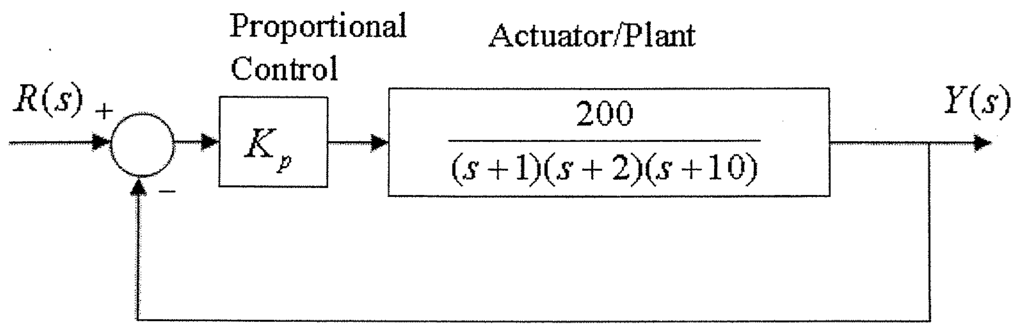

Figure Q6.1

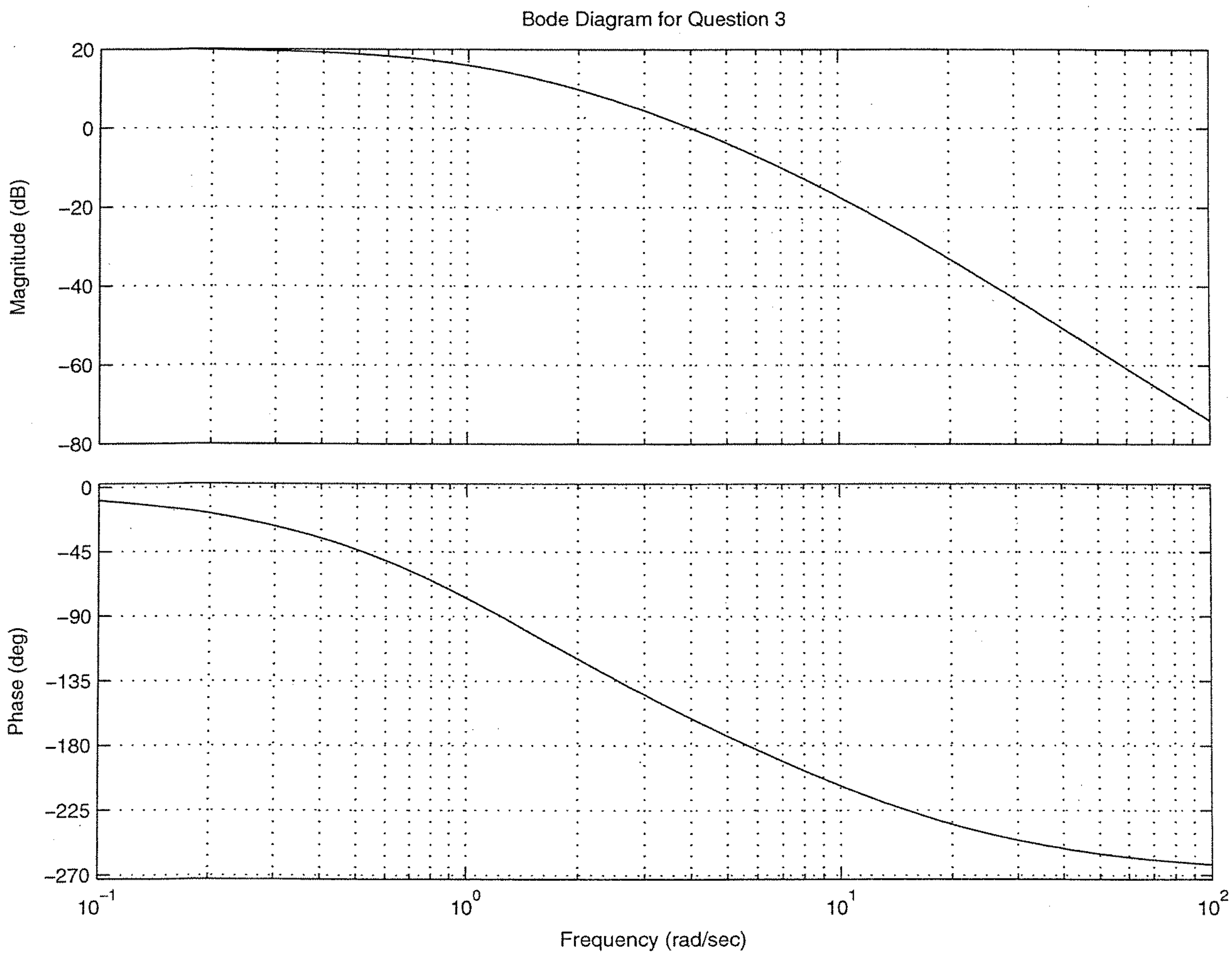

Figure Q6.2

1) **(5 marks)** Find the system Gain Margin G_m, and the system Phase Margin Φ_m, as well as the corresponding crossover frequencies. Is the closed loop system stable? What is the critical gain, K_{crit}, at which the system will be marginally stable? What is the frequency of oscillations, ω_{osc}, at that gain? Complete Table Q6.1.

Table Q6.1

$G_m =$ dB	$\omega_{cg} =$ rad/sec	$K_{crit} =$ V/V
$\Phi_m =$ degrees	$\omega_{cp} =$ rad/sec	$\omega_{osc} =$ rad/sec
put checkmark √ to indicate if the system is:	**Stable**	
	Unstable	

2) **(5 marks)** Now use the Routh-Hurwitz criterion to find the critical gain, K_{crit}, at which the system will be marginally stable, and the frequency of oscillations, ω_{osc}, at that gain. Are the results the same as from the Gain/Phase Margin calculations above? Explain any discrepancies.

3) **(5 marks)** Determine an approximate closed loop model of the system, $G_m(s)$ and its parameters, K_{dc}, ω_n, ζ , when Proportional Gain K = 1. Complete Table Q6.2.

Table Q6.2

Damping ratio of the model is:	$\zeta =$
Frequency of natural oscillations of the model is:	$\omega_n =$
DC gain of the model is:	$K_{dc} =$
Model transfer function is:	$G_m(s) =$

4) **(5 marks)** Estimate the following closed loop step response specifications: $e_{ss\%}$ - steady step error in % to a normalized unit step input, $e_{ss(ramp)}$ - steady state error to a unit ramp input, P.O. -Percent Overshoot, and $T_{settle(5\%)}$ - settling time. Complete Table Q6.3.

Table Q6.3

Percent Overshoot is:	P.O. =
Settling time is:	$T_{settle5\%} =$
Steady state error (in %) to a step is:	$e_{ss\%} =$
Steady state error to a ramp is:	$e_{ramp} =$

Question 7

An appropriate controller has to be designed for a unit feedback control system shown in Figure Q7.1. The process transfer function is the same as in Question 6 and the frequency response plots of the uncompensated open loop transfer function are as previously shown in Figure Q6.2.

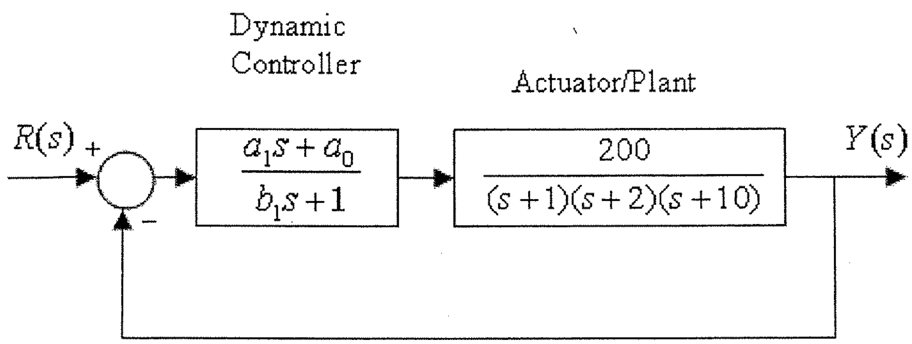

Figure Q7.1

The compensated closed loop system response requirements are that the Percent Overshoot of the normalized unit step response be no more than 20%, and that the steady state error in percent be no more than 5%. The dynamic controller configuration is to be as shown in Figure Q7.1.

1) **(4 marks)** Which parameter of the controller $G_c(s) = \dfrac{a_1 s + a_0}{b_1 s + 1}$ is affected by the steady state requirement specified above? Calculate that parameter.

2) **(4 marks)** For the remaining two parameters of the controller, what is the condition that needs to be fulfilled so that the controller can be described as either a Lead or a Lag structure? Next, the crossover frequency for Phase Margin of the uncompensated system, ω_{cp}, can be read off the Bode plot in Figure Q6.2. How will the choice of the controller structure affect its value? Will it be smaller or larger than the uncompensated value? Complete Table Q7.1.

Table Q7.1

	Lead Controller		**Lag Controller**	
Condition for the remaining controller parameters				
ω_{cp}, as compared with the uncompensated system, will be (put checkmark) √	Larger		Larger	
	Smaller		Smaller	

3) **(12 marks)** Design a Lag Controller to meet the compensated closed loop system response requirements as specified.

Question 8

Consider the feedback system shown in Figure Q8.1:

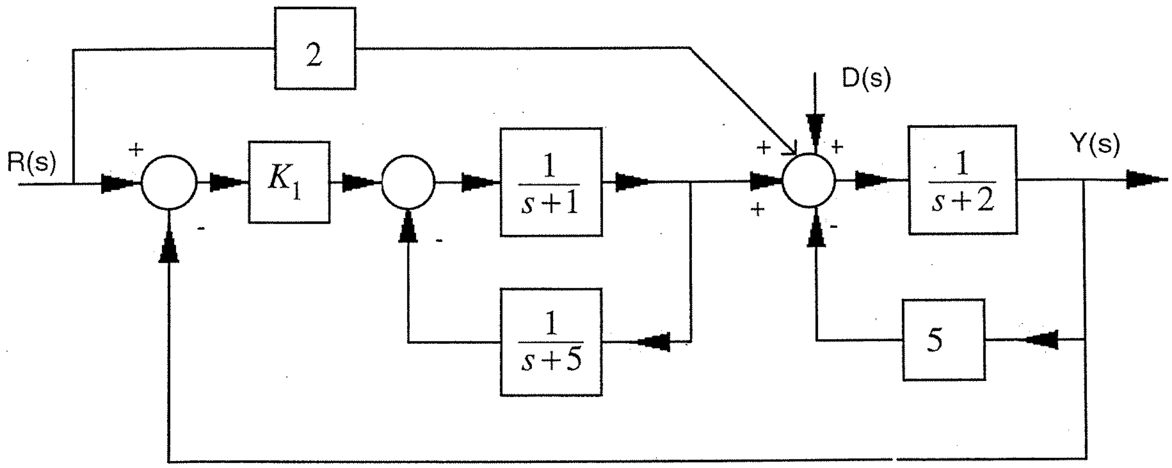

Figure Q8.1: Feedback System in Question 8

1) (**12 marks**) Apply Mason's Gain formula to obtain the transfer function of the system, $G(s) = \dfrac{Y(s)}{R(s)}$, as well as the disturbance transfer function $G_d(s) = \dfrac{Y(s)}{D(s)}$, as functions of the adjustable gain K_1.

2) (**2 marks**) Calculate the smallest gain K_1 that can be used if the steady state error of the system caused by a unit step load disturbance d(t) is to be less than 0.01.

3) (**3 marks**) Is the system stable at this value of gain K_1? Justify your answer.

4) (**3 marks**) If your answer is yes, calculate the total steady state error of the system when the reference unit step input is used and the load disturbance signal is a unit step as well.