

National Exams December 2008  
04 BS 1, Mathematics  
3 hours Duration

Notes

- 1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made
  - 2 NO CALCULATOR is permitted This is a CLOSED BOOK exam However, candidates are permitted to bring ONE AID SHEET written on both sides
  - 3 Any five questions constitute a complete paper Only the first five questions as they appear in your answer book will be marked
  - 4 All questions are of equal value
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Marking Scheme

- 1 20 marks
- 2 20 marks
- 3 20 marks
- 4 (a) 6 marks, (b) 14 marks
- 5 (a) 10 marks, (b) 10 marks
- 6 20 marks
- 7 20 marks
- 8 20 marks

- 1 Find the general solution of the differential equation  $x^2y'' - 2xy' + 2y = (1 - 2x)x^3e^{-2x}$   
Note that ' denotes differentiation with respect to  $x$
- 2 Find the general solution,  $x(t)$ , of the differential equation  $x'' + 4x = 3 \cos 2t + 4 \cos 3t$   
Note that ' denotes differentiation with respect to  $t$
- 3 Find the maximum and minimum values of  $f(x, y, z) = 4x + y^2 + 2z^2$  over the ellipsoid  $x^2 + 3y^2 + z^2 = 2$
- 4 Let  $x = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix}$  and  $A = \begin{pmatrix} 1 & 1 & 6 & -1 \\ -1 & 2 & -2 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{pmatrix}$ 
  - (a) Show that  $x$  is an eigenvector of  $A$  and find the associated eigenvalue
  - (b) Show that 3 is an eigenvalue of  $A$  and find an associated eigenvector
- 5 Let  $f(x, y, z) = x^2 + y^2 + z^2 + 2y - 3x$ , and let  $g(x, y, z) = 3x + y^2 - z^2$ 
  - (a) Find an equation for the tangent plane to the surface  $g(x, y, z) = 9$  at the point  $(3, -1, 1)$
  - (b) Find the line tangent to the intersection of the surfaces  $f(x, y, z) = 0$  and  $g(x, y, z) = 9$  at the point  $(3, -1, 1)$
- 6 Evaluate the surface integral  $\iint_S \mathbf{F} \cdot d\mathbf{S}$  where  $\mathbf{F}(x, y, z) = xz\mathbf{i} - 2y\mathbf{j} + 3x\mathbf{k}$  and  $S$  is the surface of the region bounded above by the paraboloid  $z = 4 - x^2 - y^2$  and below by the plane  $z = 0$
- 7 Find the work done by the field  $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y\mathbf{j} - z\mathbf{k}$  in moving a particle from the point  $(0, 2, 0)$  to the point  $(3\pi, 0, 2)$  along the path  $x = 6t, y = 2 \cos t, z = 2 \sin t$
- 8 Evaluate the surface integral  $\iint_S x^2yz \, dA$  where  $S$  is the portion of the cylinder  $x^2 + y^2 = 4$  with  $0 \leq z \leq 4$  and  $y \geq 0$