National Exams May 2008
04 BS 1 Mathematics
3 hours Duration

Notes

1 If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of any assumptions made

2 NO CALCULATOR is permitted This is a CLOSED BOOK exam However candidates are permitted to bring ONE AID SHEET written on both sides

3 Any five questions constitute a complete paper Only the first five questions as they appear in your answer book will be marked

4 All questions are of equal value

Marking Scheme

1 20 marks

2 20 marks

3 20 marks

4 20 marks

5 20 marks

6 (a) 5 marks (b) 5 marks (c) 5 marks (d) 5 marks

7 20 marks

8 20 marks

1 Solve the initial value problem

$$y'' - 4y = 3t + e^{2t} \qquad y(0) = 0 \;\; y'(0) = 2$$

Note that $'$ denotes differentiation with respect to t

2 Find the general solution $y(x)$ of the differential equation

$$2x^2y' \; - 5xy \; - 4y = 3x^4$$

Note that denotes differentiation with respect to x

3 Use Lagrange multipliers to find the volume of the largest box with faces parallel to the coordinate planes that can be inscribed in the ellipsoid

$$6x^2 + y^2 + 3z^2 = 2$$

4 Evaluate the line integral $\oint_C \mathbf{v} \;\; d\mathbf{r}$ where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z - 2x + y = 1$ travelled clockwise as viewed from the positive z axis and $\mathbf{v}$ is the vector function $\mathbf{v} = z + x\imath - 2y\jmath + y^2\mathbf{k}$

5 Find the equation of the plane tangent to the surface $F(x \;\; y \;\; z) = 15$ at the point P where $F(x \;\; y \;\; z) = x^2y - 2yz + z^3$ and $P = (3 \;\; 2 \;\; 1)$

6 Let P be the plane passing through the three points $(2\ 1\ 0)$ $(1\ 1\ 3)$ and $(0\ 1\ 5)$

(a) Show that these three points do not lie on the same line

(b) Give a parametric representation for the plane P

(c) Find a vector normal to the plane P

(d) Find the line of intersection between the plane P and the plane $y + z = 1$

7 Find the surface area of that portion of the surface $z = 1 - \sqrt{x^2 + y^2}$ that lies in the first octant

8 Find the volume of the solid region inside the sphere

$$x^2 + y^2 + z^2 = 4$$

and above the cone

$$z = \sqrt{x^2 + y^2}$$