Professional Engineers Ontario

National Exams- May, 2008

98-Civ-B9

Applications of the Finite Element Method

3 hours duration

Notes

1 There are 4 pages in this examination

2 If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of any assumptions made

3 This is a closed book examination One aid sheet written on both sides

4 Candidates may use one of the approved non communicating calculators

5 **Answer only TWO (2) problems out of the three proposed**
The first two problems as they appear in the answer book will be marked

6 All problems are of equal value

Problem 1 (If you select this problem, you must answer questions 1 through 10 below)

1. Explain the source of errors in finite element modelling?

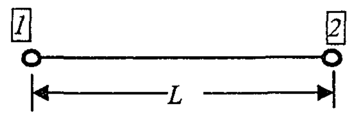
2. How is it possible to estimate the error of the results from a finite element stress analysis?

3. Select the right answer to the following statement
   The strain energy of an elastic structure calculated by the finite element method is

   a. higher than the exact value
   b. equal to the exact value
   c. lower than the exact value

4. Explain why the shape functions needed for Bernoulli's beam formulation should be a polynomial of degree 3

5. Draw the approximate shape functions for the beam element shown in the following figure



Element 1

6. Explain why there is a need for a numerical integration technique to calculate the stiffness matrix of two and three dimensional elements

7. When is it possible to adopt the assumption of plane strain for the analysis of a three dimensional structure?

8. Why are linear quadrilateral elements not good candidates for flexural dominant two dimensional elasticity problems?

9. How do you proceed if you have to analyse a buried pipe in soil subjected to a ground displacement?

10. What assumptions would be made to analyse a large concrete gravity dam subjected to seismic loads?

Problem 2

A symmetrical indeterminate truss subjected to a pair of forces is shown in Figure 2 All bars have the same rigidity EA There is no connection between members (1 3) and (2 4)

Find the displacements at all joints and the axial force acting within each bar Use the node numbering scheme shown in Figure 2

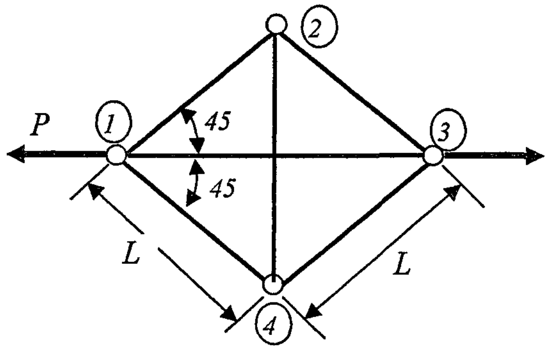
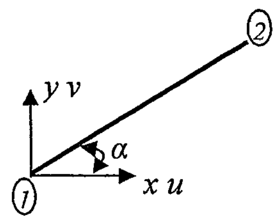


Figure 2

The stiffness matrix of a bar with orientation defined by an angle α with respect to the global reference system (x,y) is given by

$$[k]=\frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \quad C=\cos(\alpha) \quad S=\sin(\alpha)$$



The internal force in the bar is given by

$$F_{(1\,2)}=\frac{EA}{L}\begin{bmatrix} C & S \end{bmatrix}\begin{bmatrix} u_2-u_1 \\ v_2-v_1 \end{bmatrix}$$

where u_1 v_1 are the displacement of node 1 in the global reference

and u_2 v_2 are the displacement of node 2 in the global reference

Problem 3

3 1 If an undamped structure is modeled with finite elements show that its dynamic equilibrium equations can be expressed by the following system

$$[k]\{x\}+[m]\{x\}=\{f(t)\}$$

Explain each term in this equation

3 2 Explain the difference between the lumped and the consistent mass formulations in structural dynamics

3 3 Show that the fundamental frequencies of the structure are obtained by solving the following eigenvalue problem

$$\begin{cases} \text{Find}\left(\omega \quad \{q\}\right) \\ \text{such that}\left[[k]-\omega^2[m]\right]\{q\}=\{0\} \quad \text{for} \quad i=1 \quad \text{Nmode} \end{cases}$$

where ω and $\{q\}$ are the ith circular frequency and mode shape respectively

3 4 Figure 3 shows a beam with both ends fixed the density and the elastic modulus of the beam s material are denoted ρ and E respectively The moment of inertia of the beam is I A lumped mass with $m=\dfrac{4\rho AL}{35}$ and a massless elastic spring with constant $k=\dfrac{12EI}{L^3}$ are attached at the midspan as shown in Figure 3 Using the consistent mass formulation and the matrices shown below calculate the first and the second natural frequencies of the system

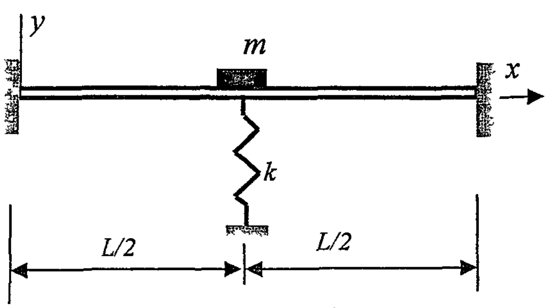


Figure 3

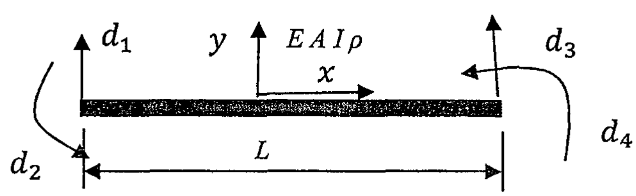


Figure 4

The stiffness and mass matrices of the beam element shown in Figure 4 are given below

$$[k]=\frac{EI}{L}\begin{bmatrix} \frac{12}{L^2} & \frac{6}{L} & -\frac{12}{L^2} & \frac{6}{L} \\ \frac{6}{L} & 4 & -\frac{6}{L} & 2 \\ -\frac{12}{L^2} & -\frac{6}{L} & \frac{12}{L^2} & -\frac{6}{L} \\ \frac{6}{L} & 2 & -\frac{6}{L} & 4 \end{bmatrix} \qquad [m]=\frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$