National Exams May 2009
04-BS-1, Mathematics
3 hours Duration

Notes

1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made

2 NO CALCULATOR is permitted This is a CLOSED BOOK exam However, candidates are permitted to bring ONE AID SHEET written on both sides

3 Any five questions constitute a complete paper Only the first five questions as they appear in your answer book will be marked

4 All questions are of equal value

Marking Scheme

1 (a) 6 marks, (b) 14 marks

2 20 marks

3 20 marks

4 20 marks

5 20 marks

6 20 marks

7 20 marks

8 20 marks

1 Consider the quadratic form $5x^2 + 24xy - 5y^2 = 13$

(a) What type of conic section is represented by the above quadratic form?

(b) Transform the quadratic form to principal axes

2 Find the general solution to the differential equation

$$y'' + 2y' - 3y = 9x^2 + e^{-3x}$$

Note that $'$ denotes differentiation with respect to x

3 Find the volume of the solid whose base is the region bounded by the parabola $y = 3x - x^2$ and the line $y = 3 - x$ and whose cross sections perpendicular to the x axis are semicircles with diameters on the x y plane

4 Find the line tangent to the intersection of the surfaces

$$6x^2 + y^2 - z = 4$$

and

$$4x^2 + y^2 + z^2 - 4y - 4z + 8 = 0$$

at the point $(1, 0, 2)$

5 Let S be the boundary of the region enclosed by the paraboloid $z = x^2 + y^2 - 2$ and the plane $z = 2$ and let

$$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + 2xyz\mathbf{j} - xz^2\mathbf{k}$$

Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ where $\mathbf{n}$ is the unit outward normal on S

6 Find the general solution to the differential equation $x^2y' + 2xy = \cos^2 x$

7 At what angle does the line represented parametrically by $x = 2 - t$, $y = t$, $z = 2 + 2t$ intersect the hyperboloid $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine

8 Find the maximum and minimum values of $f(x, y, z) = x + y - z$ over the sphere $x^2 + y^2 + z^2 = 1$