## NATIONAL EXAMINATIONS

May 2016

## 04-BS-7 MECHANICS OF FLUIDS

Three (3) hours duration

## Notes to Candidates

1. This is a Closed Book examination.
2. Exam consists of two Sections. Section $A$ is Calculative (9 questions) and Section B is Analytical (4 questions).
3. Do seven (7) questions from Section A (Calculative) and three (3) questions from Section B (Analytical). Note that the Analytical Questions do not require detailed calculations but do require full explanations.
4. Ten (10) questions constitute a complete paper. (Total 50 marks).
5. All questions are of equal value. (Each 5 marks).
6. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
7. Read the entire question before commencing the calculations and take note of hints or recommendations given.
8. Either one of the approved Casio or Sharp calculators may be used.
9. Reference information for particular questions is given on pages 9 to 11. All pages of questions attempted are to be returned with the Answer Booklet, showing diagrams generated or where readings were taken and which data was used. Candidates must write their names on these pages.
10. Constants are given on page 12.
11. Nomenclature and Reference Equations are given on pages 13 to 16 .

## SECTION A CALCULATIVE QUESTIONS

Do seven of nine questions. Solutions to these questions must be set out logically with all intermediate answers and units given.

## QUESTION 1

Refer to the diagram below which shows a horizontal pipe to which a manometer is connected. The manometer has two openings to the atmosphere.

Determine the absolute pressure P in the pipe in kPa when the manometer readings are as shown in the diagram. Refer to the Constants on Page 12 for the specific gravities of the relevant fluids.

The pipe carries water and those manometer tubes which are open to atmosphere are subject to an atmospheric pressure head equal to 10 m of water.


## QUESTION 2

For a borehole to supply an adequate amount of water it must be drilled far enough below the water table to ensure proper seepage of water from the surrounding soil. In order to check the height of the water table in a borehole 50 $m$ deep a plastic tube is inserted to the very bottom of the borehole as shown in the adjacent sketch and a small flow of air pumped down the pipe from the top. After some time a pressure gauge fitted to the top of the pipe gives a steady reading of 196 kPa gauge.
(a) Determine the depth $d$ of the water table below the surface of the ground.
(b) Sketch the profile of the water table surface near the well when water is being pumped from the well.

(5 marks)

## QUESTION 3

A concrete canal wall designed to minimize land usage has dimensions as indicated in the adjoining sketch. Determine whether or not this design will ensure that the entire base in contact with the ground is under compressive stress, that is, that the resultant force passes through the middle third of the base. Show clearly on a sketch all the forces to be considered as well as the point about which moments are taken. Refer to the Constants on Page 12 for the density of concrete.

Note: The middle third is the centre part of the base that is 2 m from each side of the base.


Hint: Consider the moments of the forces about a point at the extreme right hand side of the base.

## QUESTION 4

Preformed fibre insulation panels 2.438 mx $1.219 \mathrm{~m} \times 25 \mathrm{~mm}$ are used to insulate a building. During construction some are left lying flat on grass as shown in the adjoining sketch. If the density of a panel is $100 \mathrm{~kg} / \mathrm{m}^{3}$ calculate the minimum wind velocity (in $\mathrm{km} / \mathrm{hr}$ ) at ground level which will lift the panel and blow it away. Note that the grass under the panel reduces the air velocity to stagnation conditions.

( 5 marks )

## QUESTION 5

In an experiment to determine the head loss in a pipe system, one end of a pipe at an elevation of 6 m was connected to a reservoir of water while the other end at an elevation of 4 m was left open to the atmosphere as shown above. The pipe was 50 mm in diameter with a total length of 120 m . The reservoir contained water to a level 3 m above the pipe connection. If the measured flow rate was $0.006 \mathrm{~m}^{3} / \mathrm{s}$ determine the total head loss in the system. Assume that the pipe has a smooth entrance with no loss.

(5 marks)

## QUESTION 6

## Refer to the Examination Paper Attachments Page 9 Moody Diagram

A commercial steel pipeline is required to convey water from a storage reservoir to a local supply head tank. The length of the pipeline is 5 km and the difference in head is 120 m . Select a suitable pipe diameter to give a flow rate of $1 \mathrm{~m}^{3} / \mathrm{s}$. Assume a pipe roughness of 0.045 mm .

Return the diagram with your answer booklet to show your readings.
Hint: Set up equations of friction factor $f$ and Reynolds number Re in terms of pipe diameter $D$. Guess two or more values of $D$ in the range of 0.5 m to 0.7 m that will give points on the chart and plot these points. From a line through these points determine the pipe diameter.
( 5 marks)

## QUESTION 7



Figure $A$


Figure $B$

To ensure proper circulation of water in a swimming pool, water enters through submerged jets and spills into a trough around the sides of the pool. If one holds one's hand or a flat plate perpendicular to the jet and a short distance from the nozzle exit as in Figure A, the force of the jet can be felt. Similarly, by pushing one's hand or a flat plate against the end of the nozzle to stop the flow as in Figure B, the force due to the water pressure in the nozzle can also be felt. Consider nozzles 30 mm in diameter and a differential pressure head between the water in the supply pipe and the water in the pool of 2 m head. Are these two forces the same? Give reasons for your answer.
(a) Calculate the force due to the jet when the plate is a short distance away from the nozzle (approximately 50 mm ).
(b) Calculate the force due to the water pressure when the plate is against the nozzle and there is no flow from the nozzle.

Neglect friction effects and turbulence between the jet and the surrounding water.

## QUESTION 8

Assume that a centrifugal pump and flow system can be mathematically modelled as follows:

$$
\begin{array}{ll}
\text { Pump: } & H \\
\text { System: } & H
\end{array}=A N^{2}-B Q^{2} C^{2}+D Q^{2}
$$

where the constants and variables are as follows:

| H | $=$ | Head $(\mathrm{m})$ |
| :--- | :--- | :--- |
| Q | $=$ | Flow $\left(\mathrm{m}^{3} / \mathrm{s}\right)$ |
| N | $=$ | Pump Speed (rev/min) |
| A | $=0.000060$ |  |
| B | $=1.6$ |  |
| C | $=$ | Static Head $(\mathrm{m})$ |
| D | $=10$ |  |

For a pump speed of $900 \mathrm{rev} / \mathrm{min}$ and a static head of 20 m determine the operating point in terms of flow $Q$ and head $H$. Show the pump and system characteristics as well as the operating point in a sketch.
( 5 marks )

## QUESTION 9

## Refer to the Examination Paper Attachments Page 10 Drag on Boeing 747.

The diagram gives the drag coefficient $C_{D}$ which corresponds with the lift coefficient $C_{L}$ of a Boeing 747. These coefficients are based on the wing area which is $511 \mathrm{~m}^{2}$. If the total weight of a fully loaded Boeing 747-400 (extended cab) on reaching its cruising altitude is 320 Mg (tonnes) and its speed is Mach 0.89 determine:
(a) The coefficient of lift when flying at an altitude of 10 km where the air temperature and pressure are $-50^{\circ} \mathrm{C}$ and 26 kPa respectively.
(b) The coefficient of drag corresponding with the coefficient of lift at the conditions in (a) above.
(c) The thrust power required to maintain the speed of the aircraft.

Note that the Mach number is the speed of the aircraft divided by the speed of sound. The velocity of sound $a$ is given by $a=(k R T)^{1 / 2}$ where $k$ for air is 1.4. Show on the diagram where values were plotted and read.

## SECTION B GRAPHICAL AND ANALYTICAL QUESTIONS

Do three of four questions. These questions do not require detailed calculations but complete written explanations must be given to support the answers where descriptive answers are required.

## QUESTION 10



A barge 15 m long and 3 m wide is loaded such that its draught (depth of bottom below water surface) is 1.2 m . It sails in a long canal 5 m wide and 2 m deep. At one point the canal is taken across a valley by an aqueduct as shown in the sketch above. Determine the change in compressive force on the aqueduct pillars as the barge passes over them. Give a full explanation of your answer.

## ( 5 marks)

## QUESTION 11



Dam A


Dam B

Two small triangular dams are built on a firm flat surface as shown above. Assuming that there is no seepage under the wall but that sliding can occur, state which dam Dam A or Dam B - will be most likely to slide. Explain fully why one will be more likely to slide than the other.

## QUESTION 12

Refer to the Examination Paper Attachments Page 11 Conical Converging Nozzle.
Develop a flow net and draw streamlines for water passing through this nozzle and in the resulting jet. The given diagram must be used and the jet must be drawn well beyond the nozzle exit. Assume high velocity and consider inertia effects and the development of a vena contracta at the orifice. Show about 10 streamlines.

Return the diagram with your answer booklet.
( 5 marks )

## QUESTION 13


(a) The two sketches A and B above show streamlines of flow entering or leaving a tank through a pipe in the bottom. State which tank has the flow entering and which has the flow leaving. Give a full explanation for your answer, clarifying what fluid characteristic determines the flow in each case.


A


B
(b) The two sketches A and B above show water being discharged through sharp edged orifices under different conditions. State which one has the greater Reynolds number. Give a full explanation for your answer, clarifying what fluid characteristics determine the change in shape of the jet.
$\qquad$

Moody chart for pipe friction factor (Stanton diagram).

## QUESTION 9 DRAG ON BOEING 747



Effect on $S(x)$ and on measured drag of Boeing 747 due to fuselage modificat:
(Goodmanson and Gratzer, 1973. Courlesy of the Bocing Company)

04-BS-7 May 2016 Page 11 of 16

## NAME

## QUESTION 10 CONICAL CONVERGING NOZZLE



## 04-BS-7 MECHANICS OF FLUIDS

## GENERAL REFERENCE INFORMATION

## CONSTANTS

In engineering calculations a high degree of accuracy is seldom attained due to the neglect of minor influences or the inaccuracy of available data. For consistency in calculations however the following reasonably accurate constants should be used:

Atmospheric Pressure $\mathrm{p}_{\mathrm{o}}=100 \mathrm{kPa}$
Gravitational Acceleration $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2}$
Specific Gravity of Water $=1.00$
Specific Gravity of Glycerine $=1.26$
Specific Gravity of Mercury $=13.56$
Specific Gravity of Benzene $=0.90$
Specific Gravity of Carbon Tetrachloride $=1.59$
Density of Water $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$
Density of Sea Water $\rho=1025 \mathrm{~kg} / \mathrm{m}^{3}$
Density of Concrete $\rho=2400 \mathrm{~kg} / \mathrm{m}^{3}$
Density of Air $\rho=1.19 \mathrm{~kg} / \mathrm{m}^{3}$ (at $20^{\circ} \mathrm{C}$ ), $\rho=1.21 \mathrm{~kg} / \mathrm{m}^{3}$ (at $15^{\circ} \mathrm{C}$ )
Absolute Viscosity of Water $\mu=1.0 \times 10^{-3} \mathrm{Ns} / \mathrm{m}^{2}$
Absolute Viscosity of Air $\mu=1.8 \times 10^{-5} \mathrm{Ns} / \mathrm{m}^{2}$
Surface Tension of Water $\sigma=0.0728 \mathrm{~N} / \mathrm{m}$ (at $20^{\circ} \mathrm{C}$ )
Specific Heat of Water $\mathrm{c}_{\mathrm{p}}=4.19 \mathrm{~kJ} / \mathrm{kg}^{\circ} \mathrm{C}$
Specific Heat of Air $\mathrm{C}_{\mathrm{p}}=1005 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{C}$
Specific Heat of Air $\mathrm{c}_{\mathrm{p}}=718 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{C}$
Gas Constant for Air R = $287 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{K}$
Gas Constant for Helium R $=2077 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{K}$
Gas Constant for Hydrogen R $=4120 \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{K}$

## NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)

| a | Width | m |
| :---: | :---: | :---: |
| A | Flow area, Surface area | $\mathrm{m}^{2}$ |
| CV | Calorific value | $\mathrm{J} / \mathrm{kg}$ |
| $\mathrm{C}_{\mathrm{p}}$ | Specific heat at constant pressure | $\mathrm{J} / \mathrm{kg}^{\circ} \mathrm{C}$ |
| b | Width | m |
| D | Diameter | m |
| E | Energy | J |
| F | Force | N |
| g | Gravitational acceleration | $\mathrm{m} / \mathrm{s}^{2}$ |
| h | System head | m |
| $h_{L}$ | Head loss | m |
| H | Pump or turbine head | m |
| 1 | Moment of inertia | $\mathrm{m}^{4}$ |
| k | Ratio of specific heats |  |
| k | Loss coefficient |  |
| K | Constant |  |
| L | Length | m |
| m | Mass | kg |
| M | Mass flow rate | kg/s |
| N | Rotational speed | $\mathrm{rev} / \mathrm{s}$ |
| p | Pressure | $\mathrm{Pa}\left(\mathrm{N} / \mathrm{m}^{2}\right)$ |
| P | Power | W (J/s) |
| q | Specific heat | $\mathrm{J} / \mathrm{kg}$ |
| Q | Flow rate | $\mathrm{m}^{3} / \mathrm{s}$ |
| r | Radius | m |
| R | Specific gas constant | J/kg K |
| T | Temperature | K |
| U | Blade velocity | $\mathrm{m} / \mathrm{s}$ |
| v | Specific volume | $\mathrm{m}^{3} / \mathrm{kg}$ |
| V | Velocity | $\mathrm{m} / \mathrm{s}$ |
| V | Volume | $\mathrm{m}^{3}$ |
| w | Specific work | $\mathrm{J} / \mathrm{kg}$ |
| W | Work | J |
| y | Depth | m |
| z | Elevation | m |
| $\eta$ | Efficiency |  |
| $\mu$ | Dynamic viscosity | $\mathrm{Ns} / \mathrm{m}^{2}$ |
| v | Kinematic viscosity | $\mathrm{m}^{2} / \mathrm{s}$ |
| $\rho$ | Density | $\mathrm{kg} / \mathrm{m}^{3}$ |
| $\sigma$ | Surface tension | $\mathrm{N} / \mathrm{m}$ |
| T | Thrust | N |
| T | Shear stress | $\mathrm{N} / \mathrm{m}^{2}$ |

## REFERENCE EQUATIONS

Equation of State

$$
\begin{aligned}
& p v=R T \\
& p=\rho R T
\end{aligned}
$$

Universal Gas Law

$$
p v^{n}=\text { constant }
$$

Compressibility

$$
\beta=-\Delta / V \Delta p
$$

Viscous Force and Viscosity

$$
\begin{aligned}
& \mathrm{F}=\mu \mathrm{A} d u / \mathrm{dy} \\
& \mu=\tau /(\mathrm{du} / \mathrm{dy}) \\
& v=\mu / \rho
\end{aligned}
$$

Capillary Rise and Internal Pressure due to Surface Tension
$\mathrm{h}=(\sigma \cos \theta / \rho \mathrm{g}) \times($ perimeter $/$ area $)$
$\mathrm{p}=2 \sigma / \mathrm{r}$
Pressure at a Point

$$
\mathrm{p}=\rho \mathrm{gh}
$$

Forces on Plane Areas and Centre of Pressure
$\mathrm{F} \quad=\quad \rho g y_{c} \mathrm{~A}$
$y_{p}=y_{c}+I_{c} / y_{c} A$
Moments of Inertia
Rectangle: $\mathrm{I}_{\mathrm{c}}=\mathrm{b} \mathrm{h}^{3} / 12$
Triangle: $\quad \mathrm{I}_{\mathrm{c}}=\mathrm{b} \mathrm{h}^{3} / 36$
Circle: $\quad I_{c}=\pi D^{4} / 64$
Surface Area of Solids

$$
\text { Sphere: } \quad A=\pi D^{2}
$$

Volumes of Solids
Sphere: $\quad V=\pi D^{3} / 6$
Cone: $\quad V=\pi D^{2} h / 12$
Spherical Segment: $V=\left(3 a^{2}+3 b^{2}+4 h^{2}\right) \pi h / 2 g$
Continuity Equation

$$
\rho_{1} V_{1} A_{1}=\rho_{2} V_{2} A_{2}=M
$$

## General Energy Equation

$$
\begin{aligned}
& p_{1} / \rho_{1} g+z_{1}+V_{1}^{2} / 2 g+q_{\text {in }} / g+w_{\text {in }} / g \\
& =p_{2} / \rho_{2} g+z_{2}+V_{2}^{2} / 2 g+h_{L}+q_{\text {out }} / g+w_{\text {out }} / g
\end{aligned}
$$

Bernoulli Equation

$$
p_{1} / \rho g+z_{1}+V_{1}^{2} / 2 g=p_{2} / \rho g+z_{2}+V_{2}^{2} / 2 g
$$

Momentum Equation
Conduit: $\quad F_{R}=p_{1} A-p_{2} A-M\left(V_{2}-V_{1}\right)$
Free Jet: $\quad F_{R}=-\rho Q\left(V_{2}-V_{1}\right)$

## Flow Measurement

Venturi Tube: $\quad Q=\left[C A_{2} /\left\{1-\left(D_{2} / D_{1}\right)^{4}\right\}^{1 / 2}\right][2 \mathrm{~g} \Delta h]^{1 / 2}$
Flow Nozzle: $\quad Q=K A_{2}[2 \mathrm{~g} \Delta \mathrm{~h}]^{1 / 2}$
Orifice Meter: $\quad \mathrm{Q}=\mathrm{KA}_{0}[2 \mathrm{~g} \Delta \mathrm{~h}]^{1 / 2}$
Flow over Weirs
Rectangular Weir: $Q=C_{d}(2 / 3)[2 \mathrm{~g}]^{1 / 2} \mathrm{LH}^{3 / 2}$
Power
Turbomachine: $\quad P=\rho g Q H$
Free Jet: $\quad P=1 / 2 \rho Q V^{2}$
Moving Blades: $\quad P=M \Delta V U$

## Aircraft Propulsion

$$
\begin{aligned}
F_{\text {thrust }} & =M\left(V_{\text {jet }}-V_{\text {aircraft }}\right) \\
P_{\text {thrust }} & =M\left(V_{\text {jet }}-V_{\text {aircraft }} V_{\text {aircraft }}\right. \\
E_{\text {jet }} & =1 / 2\left(V_{\text {jet }}^{2}-V_{\text {aircraft }}\right) \\
P_{\text {jet }} & =1 / 2 M\left(V_{\text {jet }}^{2}-V_{\text {aircraft }}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{E}_{\text {fuel }}=C V_{\text {fuel }} \\
& \mathrm{P}_{\text {fuel }}=\mathrm{M}_{\text {fuel }} C V_{\text {fuel }} \\
& \eta_{\text {thermal }}=\mathrm{P}_{\text {jet }} / \mathrm{P}_{\text {fuel }} \\
& \eta_{\text {propulsion }}= \\
& P_{\text {thrust }} / P_{\text {jet }}=2 \mathrm{~V}_{\text {aircraft }} /\left(\mathrm{V}_{\text {jet }}+\mathrm{V}_{\text {aircraft }}\right) \\
& \eta_{\text {overall }}=\eta_{\text {thermal }} \times \eta_{\text {propulsion }}
\end{aligned}
$$

Wind Power

$$
\begin{aligned}
& P_{\text {total }}=1 / 2 \rho A_{T} V_{1}{ }^{3} \\
& P_{\max }=8 / 27 \rho A_{T} V_{1}^{3} \\
& H_{\max }=P_{\max } / P_{\text {total }}=16 / 27
\end{aligned}
$$

Reynolds Number

$$
\operatorname{Re}=d \vee \rho / \mu
$$

Flow in Pipes

| $h_{\mathrm{L}}$ | $=f(\mathrm{~L} / \mathrm{D})\left(\mathrm{V}^{2} / 2 \mathrm{~g}\right)$ |  |
| :--- | :--- | :--- |
| $D_{\mathrm{e}}$ | $=4$ (flow area) $/$ (wetted perimeter) |  |
| $D$ | $=D_{\mathrm{e}}$ | for non-circular pipes |
| L | $=L_{\text {total }}+\mathrm{L}_{\mathrm{e}}$ | for non-linear pipes |
| $(\mathrm{L} / \mathrm{D})$ | $=35 \mathrm{k}$ | for Re $\sim 10^{4}$ |

Drag on Immersed Bodies

$$
\begin{array}{ll}
\text { Friction Drag: } & F_{f}=C_{f} 1 / 2 \rho V^{2} B L \quad(B=\pi D) \\
\text { Pressure Drag: } & F_{p}=C_{p} 1 / 2 \rho V^{2} A \\
\text { Total Drag: } & F_{D}=C_{D} 1 / 2 \rho V^{2} A \\
\text { Aircraft Wing: } & F_{L}=C_{L} 1 / 2 \rho V^{2} A_{\text {wing }} \\
\text { Aircraft Wing: } & F_{D}=C_{D} 1 / 2 \rho V^{2} A_{\text {wing }}
\end{array}
$$

Karmen Vortex Frequency

$$
f \approx 0.20(V / D)(1-20 / R e)
$$

